Audit Fee Theory and Estimation: A Consideration of the Logarithmic Audit Fee Model

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ABSTRACT: Regressing the natural logarithm of fees on a set of predictor variables, including the natural logarithm of assets, has become the de facto standard functional form for estimating audit fees. We demonstrate that this represents a multiplicative model of fees in which all the predictor variables interact and where predicted coefficients represent elasticities; constant elasticity between fees and assets, and linearly increasing elasticity between fees and the other predictors. We show that the actual elasticities do not exhibit these properties, but that regressing by year and size partitions improves the estimation, greatly increases the explanatory power of the model, and produces residuals uncorrelated with size. We also provide examples of how the use of partitions can influence the inferences drawn from past studies.

JEL classification: M40, M42

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1. Introduction

The logarithmic audit fee model that associates logged audit fees with logged assets and other predictor variables, first adopted by Francis (1984), has become the accepted standard in the accounting literature. This paper investigates the assumptions and interpretations of the model and highlights a number of potential concerns and sources of error inherent in its use. Our goal is to broaden the understanding of the current logarithmic model specification, demonstrate empirical methodologies that improve its use, and to suggest various avenues of research that might improve audit fee model estimation and specification. In particular, we focus on two main topics.

First, on the empirical front, we demonstrate a number of issues which should be considered when developing and interpreting the results of audit fee models. We illustrate that the high explanatory power (sometimes above 80%) generated using the logarithmic model on a pooled sample applies only to the log of fees, and is in fact much lower (only around 50%) for actual (unlogged) fees. Hence, researchers should exercise care to specifically state that they are explaining variation in the log of fees, not the variation in fees. Additionally, most of the model’s predictive ability is due solely to size, with the other predictors explaining only a small fraction of the total variation. We show that the predictive power of the model can, however, be significantly increased by estimating fees in year and size partitions. Additionally, estimating the model partitioned by size quintiles or deciles prevents the model’s residuals from being correlated with size and eliminates the misclassification of firms with extreme abnormal fees.
Our second major focus is on the form and assumptions of the model itself. We demonstrate that the logarithmic audit fee model implicitly represents a multiplicative functional form with specific elasticity assumptions, which may or may not correspond well to the actual associations between the variables. The perspective that the coefficients in the logarithmic model are elasticities, and the attendant implications, have seldom been addressed in the literature. Simunic (1980) computed the elasticity between fees and company size to determine an appropriate power function for size, but did not employ a logarithmic fee estimation model.

The multiplicative functional form of the logarithmic model has several important implications for the relationship between audit fees and their determinants. First, it assumes that the elasticity of fees with respect to assets is constant over the range of assets. We demonstrate that this assumption is not correct. We discuss the fact that the non-constant association between fees and company size is often not considered in studies, and when it is, it is almost exclusively addressed as a robustness test with the sample cut at the median of assets. As our results demonstrate, however, a sample median split does not match the actual variation in coefficients across asset partitions. Second, we show that the current model assumes that all other audit fee determinants affect the magnitude of audit fees in an exponentially increasing manner, or with linearly increasing elasticity. We demonstrate that for a number of common predictor variables, neither constant nor linearly increasing elasticity appear to hold. The current form of the model therefore does not seem to accurately represent the economic intuition researchers intend when including and interpreting these variables. Third, the apparent mismatch between the model form and the actual behavior of the data results in heteroskedasticity from misspecification, which we demonstrate can be substantially reduced by estimating the model in partitions where the elasticity is relatively constant. Fourth, we show that the coefficients in the logarithmic
model are marginal effects, representing complex interaction terms with all the other predictors. This, plus the fact that there exists non-linear variation in predictors across subsets of firms, has potential implications for the conclusions drawn from the coefficients of the logarithmic model in past literature, and presents an interesting avenue of research for future studies. We provide examples of how the use of partitions can influence the inferences drawn from past studies.

In summary, this study provides an in-depth examination of the theoretical underpinnings of, and empirical issues associated with, the logarithmic audit fee model that is currently the standard in accounting research, and is similar in purpose to such papers as Hay et al. (2006) and Lennox et al. (2012). We discuss a number of empirical issues, such as running estimations by year and size partitions, which can significantly improve the predictive ability of the model and eliminate the residuals’ correlation with size. We also highlight areas, such as the non-constant values of various predictor variables across subgroups, which may prove fruitful areas for future research. Finally, we briefly discuss possible avenues of investigation which could lead to a more theoretically precise and well understood model of audit fees.

The remainder of the study is organized as follows. Section 2 provides an integrated discussion of audit fee theory, empirical estimation, and implications of the functional form of the fee models. Section 3 illustrates how applying a more accurate estimation methodology can yield interesting new insights to previously published results, and discusses implications for future research. Section 4 concludes the study.

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1 Hay et al. (2006) provides an excellent meta-analysis of the audit fee literature with the express purpose of providing a firm foundation from which future research can proceed. Lennox et al. (2012) study the properties and applicability of selections models and states in the abstract that, “A survey of 75 recent accounting articles in leading journals reveals that many researchers implement the technique [selection models] in a mechanical way with relatively little appreciation of important econometric issues and problems surrounding its use.” Likewise, our paper presents an in-depth examination of the often mechanically applied audit fee model itself so that researchers can clearly understand its assumptions and limitations, and from that baseline better use the model and move forward in developing alternative models.
2. Theory of fees and the functional form of the fee model

Functional form of the audit fee model

Audit fee studies attempt to associate audit fees with a set of predictor variables. We begin by providing a profile of the association between audit fees and their strongest predictor, total assets. Figures 1a and 1b graphically illustrate the associations between fees and assets, and log of fees and log of assets, respectively. The figures use fee data from Audit Analytics and asset data from Compustat, and include data from years 2000 through 2006. We restrict both figures only to include observations where fees are less than $20 million and assets less than $60 billion to facilitate ease of reading the graphs. Without the restriction Figure 1a becomes difficult to read, with a majority of the observations clustered in the lower left corner of the graph. We impose no such restrictions on any of the tests reported in the study. Figure 1a shows no clear linear relation between fees and assets. Although observations do appear to follow a rough curvature, the precise functional form of the relation is not obvious. Small observations dominate the data set, even with the restriction we imposed on the graph. In fact, we can restrict observations to only the lower half of both fees and assets, and the same pattern, or absence of pattern, still holds.

[Insert Figures 1a and 1b about here]

Figure 1b graphs the natural log of audit fees against the natural log of assets. A much clearer pattern emerges with these transformations compared to the raw variables in Figure 1a. One could almost envision fitting a reasonable line to the data with just a pencil and ruler. Even though this graph appears to have a slight bow rather than being a straight line, the figure does illustrate why fee models in natural log form tend to perform well, at least in terms of $R^2$. Since the log transformation severely compresses the data, the variance in the data is also severely
compressed. The deflation of variance serves to highlight any underlying association between
the variables.

Absent either a linear relation between the dependent and independent variables or a
reasonable specification of the form of a nonlinear association, estimating audit fees requires
either the use of nonlinear estimation techniques, or else techniques to create a linear estimate.
The standard audit fee regression adopts the latter approach by taking the natural logarithm of
fees as the dependent variable, the natural logarithm of total assets as the main predictor variable,
and then adding other predictor variables to the model. We refer to this as the logarithmic
specification of the audit fee model throughout our study. The logarithmic model has become
the de facto standard form of the audit fee model, as Hay et al. (2006: 146) point out in their
meta-analysis:

Regardless of the purpose, a common methodology has developed for
examining the determinants of audit fees that has been used in well over 100
published journal articles. Typically, an estimation model is developed by
regressing fees against a variety of measures surrogating for attributes that are
hypothesized to relate to audit fees, either negatively or positively. The model
typically takes the following form:

\[ \ln f_i = b_0 + b_1 \ln A_i + \sum b_k g_{ik} + \sum b_e g_{ie} + e_i, \]

where \( \ln f_i \) is the natural log of the audit fee, \( \ln A_i \) is the natural log of a size
measure (usually total assets), and \( g_{ik} \) and \( g_{ie} \) are two groups of potential fee
drivers. Most papers using this approach have addressed one (or a few)
specific independent variable(s), so the resulting regression model is usually
presented as a series of control variables \( (g_{ik}) \) that have been shown to be
significant in prior studies, plus the experimental variables \( (g_{ie}) \) that are being
added.

A review of audit fee literature from 2000 through 2012 in major journals\(^2\) suggests that fees
continue to be estimated nearly exclusively using this technique (e.g. Defond et al., 2000; Menon

\(^2\) The Accounting Review, the Journal of Accounting Research, the Journal of Accounting and Economics,
Contemporary Accounting Research, Auditing: A Journal of Practice and Theory, the Journal of Business Finance
and Williams, 2001; Ferguson and Stokes, 2002; Whisenant et al., 2003; Johnstone et al., 2004; Francis et al., 2005; Omer et al., 2006; Basioudis and Francis, 2007; Bell et al., 2008; Choi et al., 2009; Charles et al., 2010; Taylor, 2011; Numana and Willekens, 2012). Of the 62 studies we identified, 59 employ the standard log model. Very few of these studies address the size issues we discuss later in our paper even in sensitivity tests, and only three do anything beyond simple median tests, which we illustrate are insufficient. The other issues we address in the paper are almost never addressed or even mentioned in the studies.

Although not widely recognized in auditing research, the natural log transformation of the dependent variable implies a multiplicative specification for fees. We are aware of two studies where this has been discussed in the audit production literature: tangentially in Okeefe et al. (1994) on page 246 and footnote 5, and more specifically in Bell et al. (1994). The audit fee literature does not consider the implications of those studies, and fails to either acknowledge or address the issue, other than in Simunic’s (1980) seminal audit fee study. To illustrate, consider a highly simplified version of the audit fee estimation model:

\[
\text{LN(Audit Fees}_{it} = \text{LN(} \alpha \text{)} + \beta_1 \text{LN(Total Assets}_{it} + \beta_2 \sqrt{\text{Segments}}_{it} + \text{LN(} \varepsilon_{it} \text{)}, \tag{1}
\]

which researchers more commonly write in shorthand form

\[
\text{LN(Audit Fees}_{it} = \beta_0 + \beta_1 \text{LN(Total Assets}_{it} + \beta_2 \sqrt{\text{Segments}}_{it} + \eta_{it},
\]

where \( \beta_0 \) and \( \eta \) are simply the natural logarithms of \( \alpha \) and \( \varepsilon \), respectively (or alternatively, \( \alpha = e^{\beta_0} \) and \( \varepsilon = e^\eta \)). Note that the model in Equation 1 is implicitly a logarithmic transformation of the following multiplicative model:

\[
\text{Audit Fees}_{it} = \alpha \text{Total Assets}_{it}^{\beta_1} e^{\frac{1}{2} \beta_2 \text{Segments}_{it} \varepsilon_{it}} \tag{2}
\]
Next we consider how the multiplicative model of audit fees relates to the theory underlying audit pricing.

**Form of the model in relation to audit pricing theory**

The most complete development of a theory for audit pricing was provided by Simunic (1980), who recognized that external audit fees are simply a market clearing quantity \( (q) \) and price \( (p) \) pair, where quantity represents labor hours and price represents an average hourly billing rate. Assuming that any fixed portion of fees is minimal relative to the overall fee, audit fees can be described by the simple equation, \( Audit \ Fees = pq \). Interestingly, although both Simunic (1980) and Francis (1984) provide outstanding discussions of the predictor variables in their models, neither study directly relates the form of their model back to this simple specification.

Audit fees are observable, but neither \( p \) nor \( q \) is observable without access to proprietary internal firm data. Ideally, we would separately model \( p \) and \( q \). However, current audit fee theory has not developed sufficiently to allow \( p \) and \( q \) to be separately modeled, so existing audit fee models jointly estimate an unobservable price and quantity pair, \( \hat{p}q \). Whether the error in this joint estimation is independent of the level of fees is unclear, as is the precise form of the error term. If the expected magnitude of the error is constant across fee levels, then the traditional OLS regression error specification is appropriate, and the estimation of audit fees takes the form \( Audit \ Fees = \hat{p}q + e \). This equation is intrinsically nonlinear, and difficult to estimate without applying nonlinear techniques. On the other hand, if the magnitude of the estimation error is not constant, but increases as fees increase (i.e., approximately a constant percentage error), then a multiplicative specification for fees, \( Audit \ Fees = \hat{p}qe \), is appropriate.
This equation is also nonlinear, but it can be converted to a linear equation by transforming each of the variables to their natural logs: \( LN(Audit\ Fees) = LN(\tilde{p}q) + LN(e) \).

This equation resembles the form of the logarithmic audit fee model, with the major difference being that size is the only predictor variable that enters the model in log form. The remaining variables appear jointly as a vector of exponentiated predictor variables, as illustrated previously (Equations 1 and 2). Finally, note that the multiplicative model is not easily tailored to allow for fixed audit costs, which while not generally considered in fee studies, may in fact be nontrivial for small audit clients. Unfortunately, we have little theory to guide us in specifying the correct functional relation between either price or quantity and the various predictor variables appearing in the literature. Constructing and defending a specific functional form is beyond the scope of our present study, but we discuss implications of the current standard specification. Specifically, we show that interpreting the associations in the log-log model is more complex than simply examining the regression coefficients.

**Complexities in interpreting the coefficients**

For purposes of illustration, we limit the discussion to five common variables in current fee models: size (Total Assets), the quick ratio (QUICK), return on investment (ROA), audit firm size (AUDSIZE = big, small), and whether the company is subject to SOX 404 reporting requirements (SOX). In natural log form this fee model would be written as follows: \(^3\)

\[
LN(Audit\ Fees) = \beta_0 + \beta_1 LN(Total\ Assets) + \beta_2 QUICK + \beta_3 ROA + \beta_4 AUDSIZE + \beta_5 SOX + \phi,
\]

\[ (3a) \]

\(^3\) We emphasize that we have chosen this set of variables specifically to illustrate how interpreting the coefficients in the model is more complex than generally represented in the literature. We do not suggest that this subset of variables represents a fully specified model of audit fees. On the contrary, the estimation model that we use in the study (Equation 5) incorporates twenty four variables, in addition to industry and year indicator variables.
where \( \text{AUDSIZE} \) takes the value of 1 for Big-5 firms, and 0 otherwise. This model implies the following functional form for fees:

\[
\text{Audit Fees} = e^{b_0} \times \text{Total Assets}^{b_1} \times e^{b_2 \times \text{QUICK}} \times e^{b_3 \times \text{ROI}} \times e^{b_4 \times \text{AUDSIZE}} \times e^{b_5 \times \text{SOX}} \times e^{\varphi}
\] (3b)

A comparison of Equations 3a and 3b suggests some interesting associations. To begin with, it is notable that certain coefficients can be interpreted simply as intercept shifts in the logarithmic model. For example, \( \text{AUDSIZE} \) is an indicator variable in Equation 3a, which allows an intercept shift in the log of fees for Big-5 audit firms versus all others. In Equation 3b, the variable still has a constant effect, although now the effect is multiplicative rather than additive. When \( \text{AUDSIZE} \) takes the value of zero, \( e^{b_4 \times \text{AUDITOR}} \) is \( e^0 \), which is 1. When \( \text{AUDSIZE} \) takes the value of 1, \( e^{b_4 \times \text{AUDITOR}} \) is \( e^{b_4} \), which is still a constant multiplicative effect in the model. In multiplicative form, an interpretation is that \( e^{b_4} \) represents a higher per-unit service rate charged by Big-5 auditors compared to non Big-5 auditors, holding all other factors constant.

Computationally, \( \text{SOX} \) is identical to \( \text{AUDSIZE} \). However, the interpretation that auditors charge \( \text{SOX} \) clients a relatively higher per-unit rate is not as prima facie valid as for \( \text{AUDSIZE} \). \( \text{SOX} \) engagements generally are thought to be more costly because of the additional labor required to document, test, and report on controls, rather than an increase in per-hour billing rates as implied by the current model.

Other coefficients represent more complex slope coefficients. For example, the interpretations of \( \text{QUICK} \) and \( \text{ROA} \) are quite interesting. The researcher most likely desires to specify an association between audit fees and the two ratios, but Equation 3a actually specifies an association between the log of fees and the ratios. For the quick ratio, the multiplicative effect on fees is \( e^{b_2 \times \text{QUICK}} \). As we discuss more fully in the next subsection, this equates to
assuming that the relation between fees and \( \textit{QUICK} \) is an elasticity that increases linearly as a function of the quick ratio (\( \beta_2 \textit{QUICK} \)). The quick ratio is positive and its coefficient in fee regressions is typically negative. With \( \textit{QUICK} \) being positive and \( \beta_2 \) negative, the multiplier takes the following form: 
\[
e^{-\beta_2|\textit{QUICK}|}, \quad \text{or} \quad \frac{1}{e^{\beta_2|\textit{QUICK}|}}.
\]
For a firm with zero quick ratio assets, the denominator is 1 (\( e^0 = 1 \), and increases at an increasing rate as the quick ratio increases. This means that the multiplier itself has an upper bound at 1, and decreases toward zero, at a decreasing rate as the quick ratio increases. Hence, the firms with the highest liquidity risk have a multiplier of 1, and less risky firms receive a discount from that point.

\( \textit{ROA} \) may take positive or negative values, but the coefficient in fee regressions typically is negative. A negative coefficient means that when \( \textit{ROA} \) is positive, \( e^{\beta_3 \textit{ROA}} \) has an upper bound at 1 and decreases toward zero at a decreasing rate. But when \( \textit{ROA} \) is negative, \( e^{\beta_3 \textit{ROA}} \) has a lower bound at 1 and increases toward infinity at an increasing rate. In other words, firms that have higher quick ratios or \( \textit{ROA} \) pay lower fees, and those fees decrease at a decreasing rate as the ratios increase. The principal difference between the two risk ratios is that one (\( \textit{QUICK} \)) is effectively bounded between zero and 1, while the other (\( \textit{ROA} \)) is bounded between zero and infinity. The implications of having vastly different bounds on two risk variables may be worthy of investigation in future research. Furthermore, the functional forms of both of these variables have implicit assumptions for auditors' behavior toward risk. The first assumption would seem to be fairly noncontroversial: Auditors price riskier firms higher than less risky firms. The second assumption is not necessarily unreasonable, but is also not indisputable: The risk premium auditors charge increases at an increasing rate as risk increases.

Finally, the entire model represents one of marginal effects, which can lead to potential confusion in the interpretation placed on the coefficients in the current model. Researchers may
be inclined to interpret the coefficients in the model as main effects on the variables of interest. However, as should be apparent when viewing the model in its base form, each variable is specified in an interactive association with all other variables in the model. For example, the coefficient on QUICK does not specify a main effect for the variable, but rather specifies the association it has with audit fees conditional on the simultaneous, interactive effect of all other variables in the model. More importantly, the coefficients do not specify an absolute magnitude effect on fees, but rather a relative effect. In other words, the coefficients represent elasticities, which we discuss next.

*The elasticity of size and other explanatory variables*

In a typical logarithmic model of the form:

\[
LN(Y) = LN(\alpha) + \beta_1 LN(X_1) + \beta_2 LN(X_2) + \beta_3 LN(X_3) + LN(\varepsilon)
\] (4)

the estimated coefficients are elasticities. Such models are commonly used in economics to determine price and income elasticities. Price and income, however, are simply two specific forms of elasticity. As we note more extensively in Appendix 1, elasticity more generally describes the relative change in one variable for a relative change in a related variable. Note that regression coefficients from a model such Equation 4 represent the elasticity of the dependent variable with regard to each independent variable, and these elasticities are constant across the sample since all the independent variables are log transformed.

Now consider the standard form of the logarithmic audit fee model, such as that shown in Equation 3a. In this model, the only independent variable which is log transformed is assets. The rest of the independent variables are left untransformed. Unlike the regression coefficient on a log transformed variable, the regression coefficient on an untransformed independent
variable cannot be interpreted as the elasticity, but instead represents the slope of an elasticity function that increases linearly in the independent variable. This means that the standard audit fee model assumes a constant elasticity for size, but an elasticity whose absolute magnitude is linearly increasing for all the other independent variables. For the reader interested in the details of the computations, we show the derivation of these elasticities in Appendix 1.

Whether these elasticity assumptions are appropriate deserves careful consideration by the researcher. To illustrate, we first consider a theoretical specification of the elasticity of fees with respect to company size, and then demonstrate how that theory corresponds to empirical results. We then move to a discussion of other variables in the models and their specification.

Sample and descriptive statistics

Our data are taken from a variety of sources, covering the years 2000 through 2006. All financial data come from Compustat, while audit-related data, restatements, and internal control reports are from Audit Analytics. We restrict our tests only to observations with non-missing data for all relevant variables, and also exclude any observations in the financial and insurance industries. The resulting sample of 28,326 observations constitutes the bulk of audited public companies during our sample period. Descriptive statistics are profiled in Table 1, while Table 2 shows the number of observations in each estimation year. For all tests we also exclude any observation where the studentized residual is greater than three.

[Fees and company size]

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4 Some audit fee studies also exclude utilities. The statistics we report are not sensitive to the inclusion or exclusion of utilities.
5 None of the tests we report are sensitive to the inclusion or exclusion of these observations.
Certain audit costs are relatively fixed across clients, which could lead fee elasticity to decrease with size. Likewise, it is commonly accepted that sampling theory allows for fairly large returns to scale. Hence, as client size increases, the relative change in fees for a relative change in assets could decrease. But larger clients also subject the auditor to higher engagement risk and hence might be forced to pay a risk premium. In addition, at some point the inefficiencies in coordinating large engagements could eliminate or reverse efficiency gains for very large clients. Although it seems reasonable based on these factors to assume that the size elasticity of fees might not be constant, the complex factors relating size to fees make it difficult to predict ex-ante how the elasticity might vary. To provide a perspective on these elasticities we run the Equation 1 regression, excluding the Segments variable, on a pooled sample by individual year (2000-2006). These results are shown in Table 2. We then repeat the Equation 1 yearly regressions, but take the additional step of dividing each year’s sample into quintiles, and then deciles, based on total assets. We report the elasticities (the $\beta_1$ coefficients) in Table 3.

[Insert Tables 2 and 3 about here]

Table 2 shows that the sample-wide elasticity of fees with respect to size is around 0.45 (the coefficients on $\beta_1$ in the table). Panel A of Table 3 shows that the elasticity of fees with regard to size is not constant, however, but varies with size. The elasticity is approximately 28 percent among the smallest quintile of firms, varying by year from a low of 24% to a high of 31%. The elasticity is highest in the largest quintile of firms at an average of 60%, varying from a low of 53% to a high of 67%. The elasticity in the middle three quintiles is more constant, ranging on average from 41% to 49%. The elasticity differences are even more pronounced if we move to finer asset partitions. Panel B of Table 3 shows the elasticities when the regression is run by asset decile instead of quintile. The smallest decile of firms has an average elasticity of 23%,
whereas the largest two deciles have average elasticities of 58% and 63% respectively. The middle seven deciles have fairly constant elasticity except for decile 7 which spikes at 57%, although that appears to be the result of one outlying year (2000, the first year in which fee data were available).\(^6\) We do not report other partitions in the tables, but the elasticity differences between large and small clients continues to increase as we move to finer asset size partitions. Since the sample-wide elasticity parameter in Table 2 is around 0.45, the existence of non-constant elasticity across size implies that Equation 1 overstates the impact of size on fees for small clients, and understates it for large clients.

Empirically, the potential instability of the model with regard to large versus small companies has been acknowledged in the literature (e.g., Simunic, 1980; Craswell et al., 1995; Carson and Fargher, 2007). Indeed, some recent studies have sought to control for it, generally by running regressions on samples cut at the median of company size or, on rare occasions, by running nonparametric regressions. This practice is far from consistent in the literature and, even when it is present, nearly always takes the form of a robustness check against sample-wide inferences. As Table 3 illustrates, however, the divergence from a sample mean elasticity increases as one moves further into the tails of the distribution. Hence, unless a sample-wide association is only of marginal significance, a median split on company size is unlikely to yield different inferences than a full sample regression and may still mask interesting associations in the tails of the distribution that diverge from inferences made on the entire sample. For example, the average decile-based elasticities in Table 3, panel B, range from approximately the upper thirty to upper forty percentiles over nearly 80% of the distribution (deciles 2 through 9, with

\(^6\) We also note the presence of three unusually low elasticities out of the seventy reported. These are .10 and .14 in decile 5 in the years 2001 and 2006 respectively, and .08 in decile 6 in 2003. In all three cases the elasticities are not significantly different from zero and occur in the middle deciles where there is a relatively small variation in size. In these instances the relationship is captured entirely by the statistically significant intercept term.
decile 9 trending up a bit higher). It is only in the outer 10% or so of the top and bottom of the distribution that the elasticities diverge from the overall sample means reported in Table 2.

Finally, sensitivity tests attempting to control for size generally only appear in studies seeking to establish factors associated with audit fees, rather than studies estimating normal or abnormal fees as an input into a second stage equation. Interestingly, it is in this second type of study that controlling for size is particularly important, because a failure to do so leaves computed abnormal fees correlated with size. This means that associations attributed to abnormal fees in the second stage regressions of these studies could potentially be size associations, even when size is specifically controlled for in the second stage. We illustrate this and provide empirical evidence at the end of Section 3.

**Fees and other variables in the model**

While the audit fee model assumes constant elasticity for the relation between fees and assets, it assumes linearly changing elasticity for all other control variables. The coefficients in the logarithmic model, which are the slopes of the elasticity functions, can be interpreted as the relative change in the log of fees for an absolute change in the associated variable, and as such represent growth rates. For indicator variables, where the multiplicative effect is a constant, the impact is fairly simple. Consider for example the case of AUDSIZE discussed earlier, where the coefficient is $\beta_4$. The impact of a large auditor (1) versus small auditor (0) on audit fees would be $(e^{\beta_4} - e^0)$, or $(e^{\beta_4} - 1)$. This computation is encountered with some regularity in audit research (e.g., Craswell et al., 1995; Sankaraguruswamy and Whisenant, 2009), when estimates of economic significance are applied for such dichotomous variables, and correctly interpreted as a percentage change in fees when the indicator variable takes on the value of 1. For continuous
variables, however, the impact on fees is more complex because it is dependent not only on the coefficient, but also on the value of the variable itself.

In Table 4, we consider how well the assumption of linearly changing elasticity matches empirical results. Table 4 shows the fee elasticity of several variables other than size that are often employed in fee regressions. We include only non-negative, continuous variables because those require no special treatment for a natural log transformation. The quick ratio and the ratio of debt to assets are measures of liquidity risk and debt default risk, respectively. The ratio of inventory plus receivables to assets captures asset structures heavy in two accounts traditionally associated with misstatements, while the percentage of sales from foreign operations is a measure of complexity. Finally, the market-to-book ratio is a proxy for high growth.

[Insert Table 4 and Figure 2 about here]

We first rank each variable into quintiles by year, from smallest to largest, and then regress the natural log of each variable on the natural log of fees, similar to the Equation 1 treatment of assets, by year and quintile. This regression form essentially estimates a constant elasticity for each variable of interest in each variable quintile. We average the elasticity across years, and report the average for each quintile in Table 4. Below the quintile elasticities in Table 4 we show first the average yearly coefficient on the regression using the logged dependent variable without running it by quintiles (i.e. pooled) and then the average yearly coefficient of the pooled regression with the unlogged variable as the regressor. This final coefficient is similar to what is commonly estimated in audit fee models and, as noted earlier, results in elasticity equaling the coefficient times the value of the variable. In Figures 2a and 2b we graphically illustrate the quintile elasticities of the first two variables in Table 4, the quick and debt to asset ratios, and the estimated elasticities of the pooled, unlogged regression, computed as the final coefficient in
each column of Table 4 times the mean of the variable in the given quintile. We also provide
both linear and polynomial approximations for the quintile elasticities and a linear approximation
of the pooled, unlogged regression elasticities. In short, Figures 2a and 2b visually compare the
fee elasticity of size with respect to the given variable for each quintile with the estimated fee
elasticity imposed by running a pooled regression on the unlogged variable. In Figures 2c and
2d we show graphs of average logged fees by quick and debt to asset deciles, respectively, again
the first two variables from Table 4.

Table 4 and Figure 2 illustrate several important points when considering the relationship of
fees to these variables. First, the fee elasticity with respect to the different variables is not
constant across variable quintiles. In fact, for all the variables considered the elasticity changes
sign at least once, usually going from positive in the lower quintiles to negative in the higher.
Correspondingly, Figures 2c and 2d clearly show that actual fees do in fact exhibit the
relationship with the dependent variables described by the quintile elasticities. Second, as
Figures 2a and 2b show, the change in elasticity is generally not strictly linear, but better
approximated by a quadratic function. Third, the linear estimation obtained using the unlogged
dependent variable appears to provide a poor approximation of the actual fee elasticity, masking
potentially interesting variations in the relationships between fees and the variables under
consideration across quintiles. In particular, the unlogged linear approximation requires a
constant sign for the elasticity across the entire range of the dependent variable, even though the
actual elasticity appears to change signs in certain quintiles.7

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7 For brevity of illustration, Figure 2 graphs only the first two variables from Table 4. The remaining variables
exhibit similar nonlinearity when graphed. In addition, we note that no control for size is included in Table 4 or
Figure 2. In untabulated results, we added a size control to the regressions and computed the elasticities by
dependent variable quintile. This alternative specification alters the magnitude and sign of some coefficients, but all
of the variables except book to market continue to exhibit sign changes in the quintile elasticities, and appear better
approximated by a quadratic function rather than a linear one.
These findings suggest that the economic interpretation of the effect of these variables on audit fees may in many cases be more complex than previously thought, and is a potentially fruitful area for future research. For instance, a higher quick ratio normally indicates lower liquidity risk. The upper three quintiles of the quick ratio are consistent with this interpretation, showing that fees decrease as the quick ratio increases. The lower two quintiles, however, do not exhibit this relationship, indicating the presence of confounding effects with other factors. For instance, a particularly low quick ratio may occur because there are few auditable assets, leading to a positive association between fees and the ratio in the lower quintiles. Similar arguments can be made for the observed differences in the elasticities observed in Table 4. Exploring the distributions and implications of elasticities on each of the variables is beyond the scope of the current study. We simply point out that the elasticities are neither constant nor linearly varying in the way implied by the standard audit fee model, and that the cause of the variation in elasticities could have important underlying economic implications worthy of further study. Hence, a model that holds the fee elasticity of these variables constant or linearly increasing across the sample not only contributes to misspecification, but more importantly may mask critical insights across different groups of firms.

**Heteroskedasticity**

We noted earlier that the nonlinear association between audit fees and total assets is one common reason given for the logarithmic fee specification. A second reason sometimes given for the natural log transformation is that it addresses heteroskedasticity in the residuals. Heteroskedasticity can lead to incorrect standard errors in parameter estimates. It can arise in a correctly specified model, but in general heteroskedasticity indicates a model misspecification. In simple tests of association between a predictor variable and a dependent variable, common,
well-known techniques exist for mitigating false attributions due to heteroskedasticity. However, when fee residuals are employed as predictors in a second stage regression, as discussed earlier, the usual heteroskedasticity adjustments do not correct the problem.

Early fee studies employing the log-transformation of the dependent variable (e.g., Francis 1984, Francis and Simon 1987, Simon and Francis 1988) reported that the transformation eliminated heteroskedasticity. Transforming the dependent variable into its natural logarithm can reduce or eliminate heteroskedasticity due to the form of the error term in the regression (Ratkowsky 1990). Heteroskedasticity can take any functional form, but we usually think of it as the error variance increasing as the dependent variable increases. Taking the log of a multiplicative equation linearizes it, which often, although not always, results in the error variance becoming approximately homoskedastic. In recent years the log transformation no longer appears to be successful in eliminating heteroskedasticity. To illustrate we report White Chi-Square heteroskedasticity statistics for 2000 through 2006 in Table 2. The table shows that heteroskedasticity is significant in each year. Heteroskedasticity is equally significant with a full set of predictor variables (such as in Equation 5 below), so it is not simply an artifact of using size as the only regressor in the model. Figure 3 graphs the studentized residuals against the dependent variable, log of fees. As with Table 2, the figure shows that the residuals are not constant across the sample, and even exhibit a slightly positive slope. Consistent with this result, studies in later years generally have applied heteroskedasticity corrections to standard errors of the parameter estimates, even with the log transformation in use.

[Insert Figure 3 about here]

The existence of significant heteroskedasticity in the logarithmic model suggests that the transformation does not adequately linearize the relation between fees and assets, resulting in
model misspecification. This failure appears to be due to nonconstant elasticity across the sample, which we discussed earlier. If so, then running the regression in partitions where the elasticity is more constant should improve the performance of the model. Since company size is the strongest explanatory variable in the model (see discussion on explanatory power in the next section), stabilizing the elasticity on size should provide a benefit. To test this, we replicate the regression reported in Table 2 by asset quintile and decile (not tabulated). Using asset quintiles, we have five quintiles across seven years, for a total of thirty five estimations. The White Chi-Square statistic is significant in only 8 of those regressions (22.9%) as opposed to all of the regressions reported in Table 2. Using deciles, the White Chi-Square statistic is significant in only 9 of 70 estimations (12.9%).

**Explanatory power in the model**

It has become tempting to view the specification of the audit fee model as a largely resolved issue, since the explanatory power of log form fee regressions has become quite high in recent years. However, a misspecified model, even if it has a high overall fit, may provide misleading inferences in important subsamples of the population, as we demonstrated previously. Moreover, the explanatory power for a log form dependent variable (log of audit fees) is *not* equivalent to that for the underlying variable itself (audit fees). To illustrate the difference, we run a log form regression with a large set of dependent variables found in recent literature. We initially run the regression by pooling all years and including indicator variables for each year, which is the approach typically taken in fee studies. The model is as follows, where the subscript $i$ represents each company, and $t$ represents each year (see Appendix 2 for variable definitions):
\[ \text{LN}(\text{Audit Fees})_{i,t} = a + \beta_1 \text{LN}(\text{Total Assets})_{i,t} + \beta_2 \text{AUDSIZE}_{i,t} + \beta_3 \text{AUDCHG}_{i,t} + \beta_4 \text{NONDECYR}_{i,t} \\
+ \beta_5 \text{OPINLAG}_{i,t} + \beta_6 \text{GC\_OPIN}_{i,t} + \beta_7 \text{M/B}_{i,t} + \beta_8 \text{SOX}_{i,t} + \beta_9 \text{IC\_OPIN}_{i,t} + \beta_{10} \text{QUICK}_{i,t} \\
+ \beta_{11} \text{STOCKFIN}_{i,t} + \beta_{12} \text{DEBTFIN}_{i,t} + \beta_{13} \text{INVARECA}_{i,t} + \beta_{14} \text{EX\_DISC}_{i,t} + \beta_{15} \text{DEBTA}_{i,t} \\
+ \beta_{16} \text{ROA}_{i,t} + \beta_{17} \text{LOSS}_{i,t} + \beta_{18} \text{NUMSEGS}_{i,t} + \beta_{19} \text{FOR\_PCT}_{i,t} + \beta_{20} \text{ACQ}_{i,t} + \beta_{21} \text{RESTR}_{i,t} \\
+ \beta_{22} \text{RESTATE}_{i,t} + \beta_{23} \text{ZSCORE}_{i,t} + \beta_{24} \text{AGE}_{i,t} + \text{Industry Dummies} + \text{Year Dummies} + \epsilon_{i,t} \quad (5) \]

Table 5 shows the R^2 on this regression to be 82.3%. At first pass, one might be inclined to interpret this as implying that the model explains over 82 percent of the variation in audit fees, and indeed statements to this effect are not uncommon (e.g., Omer et al. 2006: 1102; Zerni, 2012: 330), but this interpretation is incorrect. The model explains over 82% of the variation in the log of audit fees, which as we discussed earlier, has highly compressed variance relative to fees themselves.

[Insert Table 5 about here]

To examine the explanatory power for audit fees (Audit Fees), we first take the anti-log of the predicted value of the regression, and then examine the portion of explained variance on that value (Bell et al., 1994; Ramanathan 1998). We compute the explanatory power on Audit Fees as

\[ 1 - \frac{\text{SSE}_{\text{AUDFEES}}}{\text{SST}_{\text{AUDFEES}}} \],

where \( \text{SSE}_{\text{AUDFEES}} \) is defined as the sum of squared differences between the actual and predicted values of Audit Fees, and \( \text{SST}_{\text{AUDFEES}} \) is defined as the sum of squared differences between the actual value of Audit Fees and the mean of Audit Fees. Note that we are not running and comparing the results from two different models here (a regression on the log of fees versus a regression on fees), but rather illustrating that the unexplained variance in the log of fees is not synonymous with the unexplained variance in fees. As shown in the first two columns of Table 5, although the model explains 82.3% of the variation in the log of audit fees, it only explains 50.8% of the variation in the audit fees themselves.
Earlier we noted that the relation between fees and assets changes over time, so we conducted all previous regressions reported in the study by year, rather than by pooling all observations and using year indicator variables. We repeat that estimation with Equation 5 by year, omitting the year indicator variables, and report the results in the second set of columns in Table 5. It is worth noting that beyond company size (log of assets), the variables in current state-of-the-art fee models explain little of the variation in fees. Table 5 reports that the explanatory power of the full model ranges from a low of 75.7% in 2000 to a high of 86.9% in 2006. Table 2 reports that the explanatory power of SIZE alone ranges from a low of 66.5% in 2000 to a high of 79.2% in 2006. Note that the results in Table 2 do not even include industry indicator variables. In no year do the incremental improvements in explanatory power from the additional regressors exceed 10%. With the exception of 2002, the model's power to explain variation in actual audit fees (not the log of audit fees) also increases nearly monotonically, but is always substantially lower than the explanatory power on the log of audit fees. Both 2000 and 2002 are particularly low, at 34.3% and 22.0%, respectively. Only in 2005 and 2006 does the explanatory power for audit fees exceed 60%, at 67.6% and 67.1%, respectively.

The explanatory power of the model can be significantly improved by running the estimation across subsamples where the elasticity of fees with regard to size is closer to being constant. We demonstrate this by repeating the estimation of Equation 5 by year and asset quintile, and again by year and asset decile. The results are in the final four columns of Table 5. The model's explanatory power improves noticeably with finer partitions of the data, especially in the first four years. In asset quintiles, the explanatory power on Audit Fees ranges from 44.5% in 2000 to 75.3% in 2006. The explanatory power in 2000 remains low at 44.5%, but 2002 improves to 46.1%. Even with decile partitions, the explanatory power of the model for Audit Fees is only
around 70% in most years, and still is quite low in 2000, at 45.8%, although 2002 improves significantly to 73.4%.

**Sensitivity tests**

To provide the broadest set of inferences, our sample includes Big 5 and non-Big 5 audit firms, as well as auditor changes, both of which we control for using the indicator variable specification common in the literature. However, some researchers prefer to exclude non-Big 5 observations and auditor changes. To ensure that our findings are not due to a spurious correlation with auditor size or auditor change, we repeat our tests after eliminating these observations. All inferences remain consistent with those reported in the study.

We also test the sensitivity of the results to industry differences. We define industry using the classifications in Barth et al. (1999), excluding financial and insurance companies. Equation 5 follows the standard practice of estimating audit fees using industry indicator variables, but the slope coefficients themselves may vary substantially by industry. If so, then the non-constant coefficients across the sample may represent industry differences not controlled for with the standard model specification. We test the stability of the coefficients by repeating the estimations in Table 3 for each individual industry group. The untabulated results show substantial variation in coefficients by industry, but the coefficients continue to vary across partitions of the relevant variables, even within industries.

**3. Empirical implications of the model form for current and future research**

We demonstrated earlier that the elasticities of the predictor variables may vary widely across the variable ranges. Since asset size dominates the predictor variables, breaking the
sample into asset partitions can help to control for this potential size correlation. To illustrate, we consider three prominent variables in prior audit fee literature that are still under active widespread study: audit firm size, auditor change, and auditor industry specialization. We first consider audit firm size as a general topic, and then replicate specific studies on auditor change and auditor industry specialization. In conducting the replications, we are not attempting to show that prior studies report "wrong" results. Rather, we hope to demonstrate that there are very interesting subsets of the data where either the primary results do not hold or that alternative interesting results are found when allowing the coefficients to vary in a way that more accurately reflects the underlying elasticities.

[Insert Table 6 about here]

Table 6 shows the coefficients and p-values on audit firm size (AUDSIZE), estimated using Equation 5 by year and asset quintile. Extensive prior research provides evidence consistent with audit fees being higher for Big 5 audit firms (e.g., Simunic 1980; Francis 1984; Hogan 1997; Ireland and Lennox 2002; Choi et al., 2008). Table 6 provides evidence consistent with prior findings for the first three client size quintiles. The association is weaker in the fourth quintile, where the coefficients are always positive, but significant at conventional levels in only four of the seven years. In the largest quintile, the association actually flips sign in all years but 2005, and is statistically significant at conventional levels in 2000 and marginally significant in 2001. Even among the quintiles where the association is consistently positive, the magnitude of the association decreases almost monotonically with each quintile. In other words, among the largest quintile of clients, the clients with Big 5 audit firms do not pay higher fees than clients of non-Big 5 firms, and may pay lower fees. This trend is similar to subsample analyses reported in Table 5 of Choi et al. (2008), and even more specifically in Carson and Fargher (2007). Choi et
al, perform a tercile analysis using cross-country data, while Carson and Fargher perform a quintile analysis using Australian data. Both of those studies demonstrate that the association between audit fees and auditor size decreases with client size. In contrast, using U.S. data we show that not only does the association decrease, but actually becomes negative among the largest clients.

[Insert Table 7 about here]

Table 7 reports the association between auditor change and audit fees, taken from Table 4 of Ghosh and Lustgarten (2006). To simplify comparison, we use their data period from 2000 to 2003, although the inferences are the same using our full dataset from 2000 through 2006. The leftmost two columns of the table report the coefficient on auditor change (AUDCHG) using their model, and pooling the data across all five quintiles of asset size as they did. The rightmost two columns report the coefficients using Equation 5 from our study, both pooled and run separately by asset quintile. The straight replication (leftmost two columns) exactly matches that report in Ghosh and Lustgarten, (-0.09, p < 0.01). The replication using our larger model provides identical inferences pooled across asset size. Examining the association in more finely partitioned client groups, however, shows that the negative association reported in their overall result varies substantially with client size. The negative association reported in Ghosh and Lustgarten (2006) is in fact found broadly across audit research. Interestingly, the quintile partitions show that the smallest quintile of clients does not experience a fee reduction, but in fact sees a fee increase. Additionally, it appears that audit fees are not sensitive to auditor changes among the largest firms. These associations might provide a very fruitful area for further research, but are masked when we force a single slope coefficient across all client sizes.
This point is illustrated even more clearly with our final replication variable, auditor industry specialization.

[Insert Table 8 about here]

Table 8 reports the association between auditor industry specialization and audit fees, taken from Table 4 of Casterella et al. (2004). Their study uses data from 1993, which we are unable to obtain. Thus, we have utilized our sample period from 2000 through 2006. For comparison, we report the results of running their model over our sample period (the leftmost two columns of Table 8) as well as those running our larger model (the rightmost two columns). Auditor industry specialization is widely found in audit research to be positively associated with audit fees. This is reflected in the models pooling across all client sizes, both in Casterella et al. (0.053, \( p = 0.090 \)) and in our replication on more recent data using both their model (0.03, \( p = 0.0012 \)) and our model from Equation 5 (0.03, \( p = 0.0005 \)). To test sensitivity of the results to client size, they partition the sample at the median of asset size and find that the result holds below the median \( (0.097, p = 0.040 \text{ in their table}; 0.03, p < 0.02 \text{ in both models of our replication}) \), but is insignificant above the median \( (0.009, p = 0.44 \text{ in their table}; 0.00, p = .8787 \text{ and } -0.01, p = .4409 \text{ in our replication models}) \). More recently, Fung et al. (2012) apply a similar median split sensitivity test to extend the analysis to the city level, finding consistent results above and below the median.\(^8\) As we illustrated and discussed with Table 3 earlier in this study, simply splitting at the median may not always be a sufficient control because the elasticities vary substantially across asset quintiles and even deciles. Reflecting this, we repeat the test using a finer quintile partition, and see that the association between auditor industry specialization and audit fees has the expected positive sign consistently in only two of the three

\(^8\) Fung et al. (2012) also report a tercile analysis in footnote 27. As we discussed earlier and illustrated in Tables 3 and 4, terciles correspond more closely to the actual elasticities than a median split, but still fail to adequately accommodate the elasticity shifts across the full sample of firms.
smallest quintiles, with the second quintile being marginal but not significantly positive. There is no association in the fourth, while the association in the largest quintile of clients actually takes a strongly negative sign.\(^9\)\(^10\) We are hesitant to draw conclusions on this result without devoting a full study specifically to the topic, but the result may indicate that the specialist auditor adds reputation value to the small client, whereas the large client benefits more from improved audit efficiency. For our present study, the important point is that even the median split that is starting to become a standard sensitivity test in audit research may not be sufficient to identify interesting subsets of the data where either the overall result fails, or where an alternative interesting result may exist.

To illustrate the association between abnormal fees (i.e., regression residuals) and company size, we provide Figures 4a and 4b. For both figures, we run the regression specified by Equation 5 earlier, and then rank the observations by company size into twenty partitions and compute the average value of the residuals for each cell. The results are nearly identical if we group the observations into quintiles or deciles, but we have opted for illustrating the effect with a high degree of granularity. In both figures we use the non-partitioned sample as a baseline, represented by the solid line with diamond shaped markers, because this is the standard estimation technique employed in current research. The graphs show a pronounced association between company size and the average residual, producing a clear "U" shape with average

\(^9\) For completeness, we also included the client Power variable in Casterella et al. (2006) in the tests. They report an overall negative association between Power and audit fees (-0.344, p=0.02), which held in the upper half of the sample (-0.589, p=0.01) but not the smaller half (-0.078, p=0.35). Using our more recent data from 2000 through 2006, we find an overall positive association (0.117, p=0.001) driven mainly by moderate to large clients (quintiles 3 and 4), but the largest clients (quintile 5) have the negative association between Power and fees (-0.226, p=0.001), as reported in Casterella et al.

\(^10\) Carson and Fargher (2007) also replicate the specialist association, using Australian data rather than U.S. data. They find that the specialist premium is highest among the large clients, suggesting that there may be a difference between the U.S. and Australian markets among the largest clients. Alternatively, our methodology allows all coefficients to vary by company size partition, whereas their methodology only allows the coefficient on assets to vary. Since the logarithmic estimation is interactive with marginal interpretations on the coefficients, as we discussed earlier, allowing conditional flexibility on all variables is preferable.
residuals of 0.24 and 0.34, respectively, in the smallest and largest size partitions. In the middle partitions of the distribution, the average residual drops to -0.13. This clear residual pattern has significant implications for any study where abnormal fees are used either as a predictor variable or as the dependent variable where the underlying association being studied could potentially be affected by company size. Further, simply including company size as an additional control variable in the second stage regression will not correct the problem, because the residual pattern is not linear.

[Insert Figures 4a and 4b about here]

We have demonstrated that running the regressions in size partitions, while not a final solution to the model specification issue, can improve the estimations as well as yield interesting new results. Employing this technique can also improve the performance of residuals when used as abnormal fees. Figure 4a illustrates the residual performance using both quintile estimation and decile estimation. The quintile estimation is represented by the dashed line with square markers. The "U" shape disappears, and the average residual is approximately zero in all partitions except at the very extremes, where they are substantially smaller than with the full pooled regression. Estimating by size decile, the association between size and the average residual almost completely disappears, except for a very small kink in the smallest partition. In the earlier replications, we showed that a simple median estimation does not adequately capture differential size effects in association studies, but it does reduce the extent to which the residuals are associated with company size, as shown in Figure 4b (the dashed line with round markers). Nonetheless, there remains a troubling pattern both at the extremes and in the middle of the distribution, which does not exist with quintile and decile estimation.
A common, and generally powerful, technique found in capital markets studies for dealing with these parametric irregularities is to employ nonparametric tests. Normally this is done by ranking observations into deciles, and then scaling so that the coefficients can be interpreted as percentage changes. To the extent that there are classification errors between partitions, even this technique can produce misleading results, or fail to find results when in fact they do exist.

We examine these classifications in untabulated tests. We find that running the typical fee regression versus regression by decile results in a 23% misclassification rate in the smallest decile, and a 28% misclassification rate in the largest decile. Comparing against the less fine partition based on quintiles, the misclassification rates are 17% in the smallest quintile and 18% in the largest quintile. Comparing against a finer partition based on twenty size cells (i.e., 5% increments), the misclassification rates are 30% in the smallest partition and 36% in the largest partition. Whether these misclassification rates are sufficient to alter results is dependent on the specifics of a particular study, but are significant enough that studies should take them into account, at least as sensitivity tests.

4. Discussion and conclusions

Regressing the natural logarithm of fees on a set of predictor variables, including the natural logarithm of assets, has become the de facto standard functional form for estimating audit fees. This paper investigates the theory and assumptions of this logarithmic model and highlights a number of potential concerns and sources of error inherent in its use. We demonstrate that the logarithmic audit fee model implicitly represents a multiplicative functional form in which all the predictor variables interact with one another and in which the predicted coefficients represent elasticities. This model specification has various theoretical and empirical implications.
The logarithmic model assumes that the elasticity of fees with respect to company size is constant over the range of assets. We show that this assumption is not correct. To date, recognition of a misspecification with regard to company size has been sporadic in the literature. When studies do address the issue, it nearly always takes the form of a robustness check against sample-wide inferences using a median split on assets, or in rare cases, nonparametric robustness checks. A median split on assets, however, does not match the underlying variation in coefficients across company size. We demonstrate this, and show that running the prediction model by year and asset size partitions, which allows for a varying elasticity of fees with regard to size, greatly increases the model’s explanatory power, and reduces the residuals’ strong correlation with size. It also substantially reduces heteroskedasticity due to model misspecification.

The logarithmic model also assumes that all other audit fee determinants affect the magnitude of audit fees in an exponentially increasing manner, or with linearly increasing elasticity. We demonstrate that for a number of common predictor variables neither constant nor strict linearly increasing elasticity holds. The model therefore does not seem to correctly capture researchers’ economic intuition with respect to these predictors or their observed empirical relation to fees. Additionally, because the logarithmic model is in essence a multiplicative model, researchers cannot interpret the coefficients as main effects (as is commonly done) but rather must view them as complex interaction terms with all the other predictors. The fact that the effects of various predictors vary across subsets of firms has potential implications for the conclusions drawn from the coefficients in the logarithmic model in past literature, and presents an interesting avenue of research for future studies.
On the empirical front, we also demonstrate that the high explanatory power generated when predicting logged fees using the model on a pooled sample does not apply to unlogged fees. The model’s explanatory power for unlogged fees is substantially lower than for the log of fees, and is due almost entirely to the relation with company size. The fact that the other predictors explain only a small fraction of the total variation in fees suggests that there is substantial room for improving the estimation.

The object of our study is to broaden our understanding of the current audit fee model, including analyzing how well the logarithmic audit fee model matches audit pricing theory, pointing out the theoretical assumptions of the logarithmic audit fee model, and beginning to assess the validity of those assumptions. Research focusing on developing a better specified model of audit fees with more precise and methodical theoretical foundations could prove a profitable area for further research. For example, asset and income decomposition models have been productive in capital markets research, and could be useful in furthering our understanding of audit pricing. In a logarithmic fee specification, however, such decompositions are difficult to envision. As a rough substitute for decomposition, audit fee models frequently incorporate variables such as the quick ratio, or the ratio of inventory and receivables to assets. However, it is not clear that such ratios yield the same inferences as would be obtained from a true decomposition model, particularly given the complexity of interpreting such variables in the present model (see discussion in Section 2 of the study).

In addition, determining how the effects of various audit fee determinants differ across various sub-groups of firms could also provide interesting economic insights. Finally, until such

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11 We thank an anonymous referee for this suggestion.
12 Consider the most basic decomposition of assets into current and noncurrent groups: that gives TA = CA + NCA. Taking the logs of both sides yields LN(TA) = LN(CA + NCA). However, LN(CA + NCA) does not equal LN(CA) + LN(NCA). Instead, LN(CA) + LN(NCA) = LN(CA x NCA).
time as better audit fee models can be developed, this paper provides a number of empirical methods for improving the explanatory power of the logarithmic audit fee model.
References


Appendix 1
Derivation of elasticities

In Section 2 of the study we describe the coefficients in the logarithmic fee model as elasticities. Specifically, we state that the coefficient on company size (Total Assets) is a constant elasticity, while the coefficients on other dependent variables represent elasticities that increase monotonically with values of the independent variable. The interpretation of the coefficients as elasticities may not be readily apparent, so in this appendix we demonstrate the derivation of the elasticities.

An elasticity is defined as "the proportional responsiveness of one variable with respect to changes in another" (e.g., Burns and Stone 1992). In other words, it is the percentage change in one variable with respect to the percentage change in the other variable. Formally, the elasticity of a variable \( y = f(x_1, ..., x_n) \) with respect to \( x_1 \) is defined as (e.g., Varian 1992, Mas-Colell et al. 1995):

\[
\frac{\partial y}{\partial x_1} \cdot \frac{x_1}{y}
\]

In the present case, the dependent variable is the audit fee (AF). The logarithmic audit fee model specifies the relation between AF and the set of independent variables as

\[
\ln(AF) = \beta_0 + \beta_1 \ln(Total \ Assets) + \beta_2 \text{QUICK} + \beta_3 \text{ROI} + \beta_4 \text{AUDSIZE} + ... + \omega.
\]

Exponentiating both sides of the equation gives

\[
e^{\ln(AF)} = e^{\beta_0 + \beta_1 \ln(Total \ Assets) + \beta_2 \text{QUICK} + \beta_3 \text{ROI} + \beta_4 \text{AUDSIZE} + ... + \omega},
\]

which equates to

\[
AF = e^{\beta_0} e^{\ln(Total \ Assets^{\beta_1})} e^{\beta_2 \text{QUICK}} e^{\beta_3 \text{ROI}} e^{\beta_4 \text{AUDSIZE}} ... e^{\omega}.
\]

Since \( e^{\ln(Total \ Assets^{\beta_1})} \) reduces to simply \( Total \ Assets^{\beta_1} \), the expression can be rewritten as

\[
AF = Total \ Assets^{\beta_1} e^{\beta_2 \text{QUICK}} e^{\beta_0} e^{\beta_3 \text{ROI}} e^{\beta_4 \text{AUDSIZE}} ... e^{\omega} \quad (A.1)
\]

In the remainder of the appendix, we illustrate the computation of elasticities on \( Total \ Assets \) and on the quick ratio. Thus, in rewriting the equation, we re-ordered \( \beta_1 \) and \( \beta_2 \) to appear before \( \beta_0 \) simply to illustrate the elasticity on these two variables, although the order of the coefficients does not matter. Denoting the impact of the error term plus all independent variables other than \( Total \ Assets \) and the quick ratio as \( f(ROA, AUDSIZE, ..., \omega) = f(\cdot) \), we finally have
\[ AF = \text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot) \]

The elasticity of audit fees with respect to \text{Total Assets} is computed as

\[ \frac{\partial AF}{\partial \text{Total Assets}} \cdot \frac{\text{Total Assets}}{AF} \]

The partial derivative of the audit fee equation with respect to \text{Total Assets} is given by

\[ \frac{\partial AF}{\partial \text{Total Assets}} = \beta_1 \text{Total Assets}^{\beta_1 - 1} e^{\beta_2 \text{QUICK}} f(\cdot) \]

so the elasticity becomes

\[ \beta_1 \text{Total Assets}^{\beta_1 - 1} e^{\beta_2 \text{QUICK}} f(\cdot) \cdot \frac{\text{Total Assets}}{\text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot)} \]

rewritten as

\[ \frac{\beta_1 \text{Total Assets}^{\beta_1 - 1} e^{\beta_2 \text{QUICK}} f(\cdot) \text{Total Assets}}{\text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot)} = \beta_1 \text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot) \]

which simplifies to just the constant \( \beta_1 \).

The elasticity of audit fees with respect to the quick ratio and all other independent variables in the model is a bit different from the elasticity of fees with respect to \text{Total Assets}, because \text{Total Assets} enters the equation in natural log form. The quick ratio's elasticity is computed as

\[ \frac{\partial AF}{\partial \text{QUICK}} \cdot \frac{\text{QUICK}}{AF} \]

The partial derivative of the audit fee equation with respect to the quick ratio is given by

\[ \frac{\partial AF}{\partial \text{QUICK}} = \beta_2 \text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot) \]

so the elasticity becomes
\[ \beta_2 \text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot) \frac{\text{QUICK}}{\text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot)} \]

rewritten as

\[ \frac{\beta_2 \text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot) \text{QUICK}}{\text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot)} = \beta_2 \text{QUICK} \cdot \frac{\text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot)}{\text{Total Assets}^{\beta_1} e^{\beta_2 \text{QUICK}} f(\cdot)} \]

which simplifies to the monotonically increasing function, \( \beta_2 \text{QUICK} \).

Referring back to the audit fee equation A.1, it becomes apparent that the elasticities between both Total Assets and audit fees, and the quick ratio and audit fees, describe the association between audit fees and each of the independent variables. In the case of Total Assets that elasticity is constant. In the case of the quick ratio and all other independent variables, that elasticity is monotonically increasing.
## Appendix 2
### Variable definitions

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACQ</td>
<td>Indicator variable defined as 1 if the company engaged in any acquisition activities during the year, and 0 otherwise (Compustat AQP &gt; 0)</td>
</tr>
<tr>
<td>AGE</td>
<td>Company age. Defined as the number of active years in the Compustat database</td>
</tr>
<tr>
<td>AUDCHG</td>
<td>Indicator variable defined as 1 if the company changed auditors during the year, and 0 otherwise (from Audit Analytics)</td>
</tr>
<tr>
<td>AUDIT FEES</td>
<td>Audit fees for the fiscal year (from Audit Analytics)</td>
</tr>
<tr>
<td>AUDSIZE</td>
<td>Indicator variable defined as 1 if the auditor is a Big-5 firm, 0 otherwise (from Audit Analytics)</td>
</tr>
<tr>
<td>DEBTA</td>
<td>Ratio of debt to total assets (Compustat DLTT/AT)</td>
</tr>
<tr>
<td>DEBTFIN</td>
<td>Indicator variable defined as 1 if the company engaged in any debt financing during the year, and 0 otherwise (based on Compustat DLTIS &gt; 0)</td>
</tr>
<tr>
<td>EX_DISC</td>
<td>Indicator variable defined as 1 if the company reported any extraordinary or discontinued items for the year, and 0 otherwise (based on Compustat XI &gt; 0)</td>
</tr>
<tr>
<td>FOR_PCT</td>
<td>Percentage of sales from foreign operations (from Compustat segment data)</td>
</tr>
<tr>
<td>GC_OPIN</td>
<td>Indicator variable defined as 1 if the company received a modification to its audit opinion, and 0 otherwise (from Audit Analytics)</td>
</tr>
<tr>
<td>IC_OPIN</td>
<td>Indicator variable defined as 1 if the company received an adverse opinion due to material weaknesses in internal controls during the year, and 0 otherwise (from Audit Analytics)</td>
</tr>
<tr>
<td>INVARECA</td>
<td>Ratio of inventory plus receivables to total assets (Compustat (RECT + INVT)/AT)</td>
</tr>
<tr>
<td>LOSS</td>
<td>Indicator variable defined as 1 if the company reported a net loss during the year, and 0 otherwise (based on Compustat NI)</td>
</tr>
<tr>
<td>M/B</td>
<td>The company's market to book ratio at the beginning of the fiscal year (Compustat PRCC_F / (CEQ/CSHO))</td>
</tr>
<tr>
<td>NONDECYR</td>
<td>Indicator variable defined as 1 if the company has a non-December fiscal year end, and 0 otherwise (base on Compustat FYR)</td>
</tr>
<tr>
<td>NUMSEGS</td>
<td>The square root of the number of operating segments reported (from Compustat segment data)</td>
</tr>
<tr>
<td>OPINLAG</td>
<td>Number of days between the end of the company's fiscal year and the date on which the audit report is issued (from Audit Analytics)</td>
</tr>
<tr>
<td>QUICK</td>
<td>Quick ratio (Compustat (ACT - INVT) / LCT)</td>
</tr>
<tr>
<td>RESTATE</td>
<td>Indicator variable defined as 1 if the company was engaged in restatement activities during the year, 0 otherwise (from Audit Analytics)</td>
</tr>
<tr>
<td>RESTR</td>
<td>Indicator variable defined as 1 if the company engaged in any restructuring activities during the year, and 0 otherwise (based on Compustat RCP &gt; 0)</td>
</tr>
<tr>
<td>ROA</td>
<td>Return on assets (Compustat EBIT/AT)</td>
</tr>
<tr>
<td>SOX</td>
<td>Indicator variable defined as 1 if the company's controls were audited pursuant to SOX section 404, and 0 otherwise (from Audit Analytics)</td>
</tr>
<tr>
<td>STOCKFIN</td>
<td>Indicator variable defined as 1 if the company engaged in any stock financing during the year, and 0 otherwise (based on Compustat SSTK &gt; 0)</td>
</tr>
<tr>
<td>TOTAL ASSETS</td>
<td>Total assets (AT)</td>
</tr>
<tr>
<td>ZSCORE</td>
<td>Bankruptcy score from Zmijewski (1984), computed as:-4.336 -4.513<em>ROA +5.679</em>Leverage +0.004*Current Ratio. Note that a higher score indicates greater financial distress. ROA = (NI+TIE)/AT. Leverage = LT/AT. Current Ratio = ACT/LCT</td>
</tr>
</tbody>
</table>
## TABLE 1
Descriptive statistics

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Mean</th>
<th>Median</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Standard Deviation</th>
<th>25th Percentile</th>
<th>75th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUDIT FEES (in $)</td>
<td>936,013</td>
<td>272,563</td>
<td>2,000</td>
<td>42,200,000</td>
<td>2,186,986</td>
<td>111,000</td>
<td>820,000</td>
</tr>
<tr>
<td>TOTAL ASSETS (in $ millions)</td>
<td>1,904.82</td>
<td>147.62</td>
<td>0.00</td>
<td>270,634</td>
<td>8.212</td>
<td>27.24</td>
<td>758.85</td>
</tr>
<tr>
<td>Sales (in $ millions)</td>
<td>1,630.19</td>
<td>126.27</td>
<td>0.00</td>
<td>335,086</td>
<td>7.983</td>
<td>19.05</td>
<td>707.41</td>
</tr>
<tr>
<td>Market Value (in $ millions)</td>
<td>2,273.03</td>
<td>162.09</td>
<td>0.00</td>
<td>439,013</td>
<td>12.255</td>
<td>26.79</td>
<td>835.39</td>
</tr>
<tr>
<td>LN(TOTAL ASSETS)</td>
<td>18.725</td>
<td>18.810</td>
<td>6.908</td>
<td>26.324</td>
<td>2.576</td>
<td>17.120</td>
<td>20.447</td>
</tr>
<tr>
<td>ACQ</td>
<td>0.036</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.187</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AGE</td>
<td>17.106</td>
<td>12</td>
<td>1</td>
<td>57</td>
<td>13.927</td>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>AUDCHG</td>
<td>0.007</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.084</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>AUDSIZE</td>
<td>0.725</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.447</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>DEBT_TA</td>
<td>0.211</td>
<td>0.086</td>
<td>0.000</td>
<td>91.000</td>
<td>0.994</td>
<td>0.000</td>
<td>0.278</td>
</tr>
<tr>
<td>DEBT_FIN</td>
<td>0.469</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.499</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>EX_DISC</td>
<td>0.222</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.416</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FOR_PCT</td>
<td>0.136</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.234</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>GC_OPIN</td>
<td>0.117</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.322</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>IC_OPIN</td>
<td>0.027</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.163</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>INVARECA</td>
<td>0.256</td>
<td>0.217</td>
<td>0.000</td>
<td>1.000</td>
<td>0.204</td>
<td>0.086</td>
<td>0.378</td>
</tr>
<tr>
<td>LOSS</td>
<td>0.458</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.498</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>M/B</td>
<td>3.762</td>
<td>1.914</td>
<td>-4,881</td>
<td>8,061</td>
<td>109.411</td>
<td>0.959</td>
<td>3.559</td>
</tr>
<tr>
<td>NONDECYR</td>
<td>0.308</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.462</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>NUMSEGS</td>
<td>1.213</td>
<td>1.000</td>
<td>0.000</td>
<td>3.464</td>
<td>0.618</td>
<td>1.000</td>
<td>1.732</td>
</tr>
<tr>
<td>OPINLAG</td>
<td>66.595</td>
<td>62</td>
<td>0</td>
<td>1,171</td>
<td>48.450</td>
<td>44</td>
<td>77</td>
</tr>
<tr>
<td>QUICK</td>
<td>3.070</td>
<td>1.392</td>
<td>0.000</td>
<td>1,907</td>
<td>20.879</td>
<td>0.820</td>
<td>2.674</td>
</tr>
<tr>
<td>RESTATE</td>
<td>0.003</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.059</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>RESTR</td>
<td>0.220</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.415</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>ROA</td>
<td>-0.635</td>
<td>0.040</td>
<td>-695</td>
<td>5.175</td>
<td>10.090</td>
<td>-0.132</td>
<td>0.100</td>
</tr>
<tr>
<td>SOX</td>
<td>0.232</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.422</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>STOCKFIN</td>
<td>0.814</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.389</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>ZSCORE</td>
<td>11.228</td>
<td>-1.315</td>
<td>-8,208</td>
<td>37,033</td>
<td>340.389</td>
<td>-2.556</td>
<td>0.139</td>
</tr>
</tbody>
</table>

**Notes:**

Table 1 shows the basic descriptive statistics on variables used in the paper. Variable descriptions are provided in Appendix 2. The statistics are restricted to observations with non-missing data on all variables from 2000 through 2006, comprising 28,326 observations in total. The numbers of observations per year are shown in Table 2.
TABLE 2
Regression of log of fees on log of assets, by year

Model: $\ln(Audit \ Fees_i) = \ln(\alpha) + \beta_1 \ln(Total \ Assets_i) + \ln(\varepsilon_i)$,

| Year | Number of Obs | Intercept: $\ln(\alpha)$* | Parameter Estimate: $\beta_1$* | Model R-square | White Chi-Square |
|------|--------------|-----------------------------|-----------------------------|----------------|-----------------
| 2000 | 2,715        | 4.04                        | 0.431                       | 66.5%          | 19.46 (p<0.0001) |
| 2001 | 4,220        | 4.27                        | 0.423                       | 69.9%          | 49.23 (p<0.0001) |
| 2002 | 4,414        | 4.36                        | 0.428                       | 72.0%          | 47.22 (p<0.0001) |
| 2003 | 4,439        | 4.29                        | 0.441                       | 75.6%          | 58.39 (p<0.0001) |
| 2004 | 4,420        | 3.57                        | 0.498                       | 76.4%          | 22.87 (p<0.0001) |
| 2005 | 4,362        | 3.46                        | 0.514                       | 78.2%          | 35.27 (p<0.0001) |
| 2006 | 3,756        | 3.64                        | 0.507                       | 79.2%          | 32.82 (p<0.0001) |

Notes:
* Intercept and parameter estimate are significant at $p < 0.0001$ each year.

Table 2 shows the size elasticity of audit fees by year ($\beta_1$), along with the explanatory power and heteroskedasticity statistic from the regression. Note that intercepts in logarithmic fee regressions in some studies range between 9 and 11. Ours range between 3 and 5. The apparent difference is due to scaling assets as reported by Compustat prior to computing the natural log. Compustat reports assets in millions. Audit Analytics reports fees in dollars, so we scale assets to dollars as well. Since assets are only being adjusted by a fixed multiplier in either case, the unit of measure has no effect on the parameter estimates in the log form regression or on the statistical significance of the intercept, but rather only on the scale of the intercept. In other words, if we leave assets in millions as reported by Compustat, the intercept term in our regressions ranges from 9 to 11.
TABLE 3
Elasticity of fees by year and size group

Model: \( \text{ln(Audit Fees)}_i = \text{ln(} \alpha \text{)} + \beta_1 \text{ln(Total Assets)}_i + \text{ln(} \varepsilon_i \text{)}, \)

**Panel A:** Size elasticity of fees, by year and asset quintile

<table>
<thead>
<tr>
<th>Quintile</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>Average Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.31</td>
<td>0.29</td>
<td>0.24</td>
<td>0.28</td>
<td>0.25</td>
<td>0.28</td>
<td>0.30</td>
<td>0.28</td>
</tr>
<tr>
<td>2</td>
<td>0.34</td>
<td>0.34</td>
<td>0.38</td>
<td>0.45</td>
<td>0.62</td>
<td>0.71</td>
<td>0.62</td>
<td>0.49</td>
</tr>
<tr>
<td>3</td>
<td>0.52</td>
<td>0.20</td>
<td>0.28</td>
<td>0.34</td>
<td>0.46</td>
<td>0.57</td>
<td>0.48</td>
<td>0.41</td>
</tr>
<tr>
<td>4</td>
<td>0.51</td>
<td>0.53</td>
<td>0.56</td>
<td>0.49</td>
<td>0.41</td>
<td>0.42</td>
<td>0.45</td>
<td>0.48</td>
</tr>
<tr>
<td>5</td>
<td>0.57</td>
<td>0.60</td>
<td>0.67</td>
<td>0.64</td>
<td>0.62</td>
<td>0.55</td>
<td>0.53</td>
<td>0.60</td>
</tr>
</tbody>
</table>

**Panel B:** Size elasticity of fees, by year and asset decile

<table>
<thead>
<tr>
<th>Decile</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>Average Elasticity</th>
<th>Average Elasticity, with all control variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.31</td>
<td>0.28</td>
<td>0.20</td>
<td>0.18</td>
<td>0.22</td>
<td>0.19</td>
<td>0.23</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.49</td>
<td>0.28</td>
<td>0.46</td>
<td>0.48</td>
<td>0.31</td>
<td>0.58</td>
<td>0.49</td>
<td>0.44</td>
<td>0.46</td>
</tr>
<tr>
<td>3</td>
<td>0.29</td>
<td>0.44</td>
<td>0.27</td>
<td>0.38</td>
<td>0.63</td>
<td>0.80</td>
<td>0.64</td>
<td>0.49</td>
<td>0.40</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>0.36</td>
<td>0.23</td>
<td>0.47</td>
<td>0.70</td>
<td>0.58</td>
<td>0.41</td>
<td>0.43</td>
<td>0.39</td>
</tr>
<tr>
<td>5</td>
<td>0.42</td>
<td>0.10</td>
<td>0.47</td>
<td>0.52</td>
<td>0.55</td>
<td>0.65</td>
<td>0.14</td>
<td>0.41</td>
<td>0.37</td>
</tr>
<tr>
<td>6</td>
<td>0.45</td>
<td>0.24</td>
<td>0.33</td>
<td>0.08</td>
<td>0.51</td>
<td>0.65</td>
<td>0.46</td>
<td>0.39</td>
<td>0.36</td>
</tr>
<tr>
<td>7</td>
<td>0.95</td>
<td>0.59</td>
<td>0.55</td>
<td>0.54</td>
<td>0.47</td>
<td>0.41</td>
<td>0.49</td>
<td>0.57</td>
<td>0.52</td>
</tr>
<tr>
<td>8</td>
<td>0.39</td>
<td>0.55</td>
<td>0.50</td>
<td>0.39</td>
<td>0.33</td>
<td>0.30</td>
<td>0.44</td>
<td>0.41</td>
<td>0.43</td>
</tr>
<tr>
<td>9</td>
<td>0.50</td>
<td>0.62</td>
<td>0.67</td>
<td>0.72</td>
<td>0.60</td>
<td>0.47</td>
<td>0.49</td>
<td>0.58</td>
<td>0.54</td>
</tr>
<tr>
<td>10</td>
<td>0.64</td>
<td>0.63</td>
<td>0.71</td>
<td>0.60</td>
<td>0.64</td>
<td>0.55</td>
<td>0.54</td>
<td>0.63</td>
<td>0.61</td>
</tr>
</tbody>
</table>
Notes:
Table 3, panels A and B report the coefficients on $\beta_1$ from the regression model in Equation 2, by year and size group (defined by total assets). For consistency in the paper, we generally define size groups at the asset quintile level, although panel B above also reports the more finely detailed partition at the decile level. Both tables show that audit fees do not exhibit constant elasticity with regard to size, but rather exhibit lower elasticity among small clients, and higher elasticity among large clients. In addition, the final column of panel B reports the coefficient on size from the regression with a full set of control variables (Equation 5), illustrating that the trend in size elasticity of fees is the same with or without control variables.
### Table 4
Average elasticity of fees with other common non-negative continuous independent variables, by quintile of the associated variable

**Model 1**: \(\ln(Audit \ fees_i) = \ln(\alpha) + \beta_1 \ln(Variable_i) + \ln(\varepsilon_i)\)

**Model 2**: \(\ln(Audit \ fees_i) = \ln(\alpha) + \beta_1 (Variable_i) + \ln(\varepsilon_i)\)

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Quick Ratio</th>
<th>Debt to Assets</th>
<th>Inventory + Receivables to Assets</th>
<th>% of Sales from Foreign Operations</th>
<th>Market to Book Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.636</td>
<td>-0.074</td>
<td>0.303</td>
<td>0.035</td>
<td>0.320</td>
</tr>
<tr>
<td>2</td>
<td>0.039</td>
<td>0.413</td>
<td>-0.024</td>
<td>0.064</td>
<td>0.491</td>
</tr>
<tr>
<td>3</td>
<td>-0.359</td>
<td>0.271</td>
<td>0.016</td>
<td>1.192</td>
<td>0.090</td>
</tr>
<tr>
<td>4</td>
<td>-0.635</td>
<td>0.146</td>
<td>-1.067</td>
<td>1.021</td>
<td>-0.302</td>
</tr>
<tr>
<td>5</td>
<td>-0.477</td>
<td>-0.905</td>
<td>-1.524</td>
<td>-1.831</td>
<td>-0.390</td>
</tr>
</tbody>
</table>

Model 1, Pooled by Year: 0.024 0.087 0.041 0.169 -0.065

Model 2, Pooled by Year: -0.022 -0.077 -0.714 0.449 -0.002

**Notes:**
Table 4 reports the elasticity of audit fees with regard to other continuous non-negative variables commonly found in audit fee models. Model 1 represents the basic elasticity computation, which we estimate by quintile of the respective variable, and also estimate by pooling across all quintiles (Model 1, Pooled by Year). For comparison, we also show the pooled coefficient on each variable when it enters the estimation in unlogged form (Model 2, Pooled by Year). Model 2 is similar to the estimation currently used in most audit fee research and, as discussed in Section 2 of the study, imposes a linearly changing elasticity with single sign across the sample. That structure, however, does not correspond to the actual elasticities reported above and graphed in Figure 2, which do not change in a strictly linear manner and generally exhibits sign changes.
TABLE 5
Comparison of explained variance from the fee regression model

Model (Equation 5):
\[ \ln(\text{Audit Fees})_{i,t} = a + \beta_1 \ln(\text{TotalAssets})_{i,t} + \beta_2 \text{AUDSIZE}_{i,t} + \beta_3 \text{AUDCHG}{}_{i,t} + \beta_4 \text{NONDECYR}_{i,t} + \beta_5 \text{OPINLAG}{}_{i,t} + \beta_6 \text{GC\_OPIN}{}_{i,t} + \beta_7 \text{B/M}{}_{i,t} + \beta_8 \text{SOX}{}_{i,t} + \beta_9 \text{IC\_OPIN}{}_{i,t} + \beta_{10} \text{QUICK}{}_{i,t} + \beta_{11} \text{STOCKFIN}{}_{i,t} + \beta_{12} \text{DEBTFIN}{}_{i,t} + \beta_{13} \text{INVARECA}{}_{i,t} + \beta_{14} \text{EX\_DISC}{}_{i,t} + \beta_{15} \text{DEBTA}{}_{i,t} + \beta_{16} \text{ROA}{}_{i,t} + \beta_{17} \text{LOSS}{}_{i,t} + \beta_{18} \text{NUMSEGS}{}_{i,t} + \beta_{19} \text{FOR\_PCT}{}_{i,t} + \beta_{20} \text{ACQ}{}_{i,t} + \beta_{21} \text{RESTR}{}_{i,t} + \beta_{22} \text{RESTATE}{}_{i,t} + \beta_{23} \text{ZSCORE}{}_{i,t} + \beta_{24} \text{AGE}{}_{i,t} + \text{Industry Dummies} + \epsilon_{i,t} \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Pooled Sample with Year Dummies</th>
<th>By Year</th>
<th>By Year and Asset Quintile</th>
<th>By Year and Asset Decile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ln(Fees)</td>
<td>Fees</td>
<td>ln(Fees)</td>
<td>Fees</td>
</tr>
<tr>
<td>2000</td>
<td>82.3%</td>
<td>50.8%</td>
<td>75.7%</td>
<td>34.3%</td>
</tr>
<tr>
<td>2001</td>
<td>78.2%</td>
<td>46.6%</td>
<td>82.1%</td>
<td>64.9%</td>
</tr>
<tr>
<td>2002</td>
<td>79.8%</td>
<td>22.0%</td>
<td>83.3%</td>
<td>46.1%</td>
</tr>
<tr>
<td>2003</td>
<td>81.9%</td>
<td>49.4%</td>
<td>85.2%</td>
<td>69.9%</td>
</tr>
<tr>
<td>2004</td>
<td>86.1%</td>
<td>53.3%</td>
<td>88.1%</td>
<td>65.7%</td>
</tr>
<tr>
<td>2005</td>
<td>85.9%</td>
<td>67.6%</td>
<td>87.7%</td>
<td>67.9%</td>
</tr>
</tbody>
</table>

Notes:
Table 5 reports the explained variance from the full regression model (Equation 5) by year, by year and asset quintile, and by year and asset decile. The explanatory power for ln(Fees) is the r-square reported from the regression. Note that we are not comparing two models here, but rather illustrating that the unexplained variance in the log of fees is not synonymous with the unexplained variance in fees. This difference is due both to the natural compression derived from the log transformation discussed in Section 2 of the study, and the median shift discussed in Section 3. Since fees do not exhibit constant elasticity with respect to size (defined as total assets), and since size provides most of the explanatory power of the model, the table also illustrates that the variance in fees captured in the log-log regression improves as size groups become more homogeneous in the estimation.
TABLE 6
Coefficients on audit firm size (AUDSIZE) from the audit fee regression specified in Equation 5, by year and asset quintile

Model (Equation 5):
\[
\text{LN(Audit Fees)}_{i,t} = a + \beta_1 \text{LN(Total Assets)}_{i,t} + \beta_2 \text{AUDSIZE}_{i,t} + \beta_3 \text{AUDCHG}_{i,t} + \beta_4 \text{NONDECYR}_{i,t} + \beta_5 \text{OPINLAG}_{i,t} + \beta_6 \text{GC_OPIN} \_i,t + \beta_7 \text{B/M}_{i,t} + \beta_8 \text{SOX}_{i,t} + \beta_9 \text{JIC\_OPIN} \_i,t + \beta_{10} \text{QUICK} \_i,t + \beta_{11} \text{STOCKFIN}_{i,t} + \beta_{12} \text{DEBTFIN}_{i,t} + \beta_{13} \text{INVARECA} \_i,t + \beta_{14} \text{EX\_DISC} \_i,t + \beta_{15} \text{DEBTA} \_i,t + \beta_{16} \text{ROA} \_i,t + \beta_{17} \text{LOSS} \_i,t + \beta_{18} \text{NUMSEGS} \_i,t + \beta_{19} \text{FOR\_PCT} \_i,t + \beta_{20} \text{ACQ} \_i,t + \beta_{21} \text{RESTR} \_i,t + \beta_{22} \text{RESTATE} \_i,t + \beta_{23} \text{ZSCORE} \_i,t + \beta_{24} \text{AGE} \_i,t + \text{Industry Dummies} + \epsilon_{i,t}
\]

<table>
<thead>
<tr>
<th>Quintile</th>
<th>Year</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td></td>
<td>0.117</td>
<td>0.203</td>
<td>0.164</td>
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<td>0.0730</td>
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<tr>
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<td>0.312</td>
<td>0.192</td>
<td>0.062</td>
<td>0.139</td>
<td>-0.204</td>
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<td>0.0000</td>
<td>0.2503</td>
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<td>0.1860</td>
</tr>
<tr>
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<td>0.220</td>
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</tr>
<tr>
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<td></td>
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<td>0.0000</td>
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<td>0.0784</td>
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</tr>
<tr>
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<td>0.121</td>
<td>-0.033</td>
</tr>
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<td>0.0000</td>
<td>0.0000</td>
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<td>0.8417</td>
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<tr>
<td>2004</td>
<td></td>
<td>0.707</td>
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<td>-0.062</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0005</td>
<td>0.6512</td>
</tr>
<tr>
<td>2005</td>
<td></td>
<td>0.684</td>
<td>0.429</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.5142</td>
</tr>
<tr>
<td>2006</td>
<td></td>
<td>0.612</td>
<td>0.430</td>
<td>0.279</td>
<td>0.194</td>
<td>-0.057</td>
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<tr>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0041</td>
<td>0.6894</td>
</tr>
</tbody>
</table>

Notes:
Table 6 reports the coefficients on AUDSIZE ($\beta_2$) from the estimation of Equation 5 by year and asset quintile. For each year and quintile pair, the coefficient is reported on top (in boldface) and the 2-sided p-value is reported on bottom. In the lowest two asset quintiles, the coefficients on AUDSIZE show that Big 5 audit firms are significantly associated with higher fees in each year. In the third quintile, the coefficients on AUDSIZE are positive and significant at conventional levels in all years except 2001. The association is weaker in the fourth quintile, where the coefficients are always positive, but significant at conventional levels in only four of the seven years. In the largest quintile, the association actually flips sign in all years but 2005, and is statistically significant at conventional levels in 2000 and marginally significant in 2001. Further, even among the first three quintiles where the association is generally strongly positive, the magnitude of the association decreases monotonically with each quintile.
TABLE 7
Replication of association between auditor change and audit fees, from Ghosh and Lustgarten 2006, Table 4 Panel A

<table>
<thead>
<tr>
<th></th>
<th>Ghosh &amp; Lustgarten Table 4 Replication</th>
<th>Ghosh &amp; Lustgarten Table 4 Replication By Asset Quintile Using Our Equation 5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>p-value</td>
</tr>
<tr>
<td>Pooled</td>
<td>-0.09</td>
<td>0.0001</td>
</tr>
<tr>
<td>Quintile 1</td>
<td>0.06</td>
<td>0.0188</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>-0.12</td>
<td>0.0001</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>-0.08</td>
<td>0.0060</td>
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<tr>
<td>Quintile 4</td>
<td>-0.13</td>
<td>0.0001</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>-0.05</td>
<td>0.2370</td>
</tr>
</tbody>
</table>

Notes:
Table 7 reports the association between auditor change and audit fees, taken from Table 4 of Ghosh and Lustgarten (2006), using their data period from 2000 to 2003. The leftmost two columns report the coefficient on auditor change (AUDCHG) using their model, and pooling the data across all five quintiles of asset size. The rightmost two columns report the coefficients using our model from Equation 5, both pooled and run separately by asset quintile. In the interest of brevity, we have limited the table to report only the primary variable of interest, auditor change. The replications pooled across asset size using either model provide identical inferences to those reported by Ghosh and Lustgarten. Examining the association in more finely partitioned client groups however, shows that the negative association reported in their overall result, and more widely throughout the body of audit research, varies substantially with client size. Among the smallest quintile of clients, not only do the clients not experience a fee reduction, but in fact see a fee increase.
TABLE 8
Replication of association between auditor specialization and audit fees, from Casterella et al. 2004, Table 4

<table>
<thead>
<tr>
<th></th>
<th>Results using Model Specified by</th>
<th>Results using Equation 5 From</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Casterella et al. (2004)</td>
<td>Our Study</td>
</tr>
<tr>
<td></td>
<td>Coefficient</td>
<td>p-value</td>
</tr>
<tr>
<td>Pooled</td>
<td>0.03</td>
<td>0.0012</td>
</tr>
<tr>
<td>Below Median</td>
<td>0.03</td>
<td>0.0198</td>
</tr>
<tr>
<td>Above Median</td>
<td>0.00</td>
<td>0.8787</td>
</tr>
<tr>
<td>Quintile 1</td>
<td>0.03</td>
<td>0.0976</td>
</tr>
<tr>
<td>Quintile 2</td>
<td>0.02</td>
<td>0.1787</td>
</tr>
<tr>
<td>Quintile 3</td>
<td>0.05</td>
<td>0.0161</td>
</tr>
<tr>
<td>Quintile 4</td>
<td>0.00</td>
<td>0.8722</td>
</tr>
<tr>
<td>Quintile 5</td>
<td>-0.07</td>
<td>0.0019</td>
</tr>
</tbody>
</table>

Notes:
Table 8 reports the association between auditor industry specialization and audit fees, taken from Table 4 of Casterella et al. (2004). Their study uses data from 1993, which we are unable to obtain. Thus, we have utilized our sample period from 2000 through 2006. For comparison, we report the results of running their model over our sample period (leftmost two columns) as well as those running our larger model (rightmost two columns). Both in Casterella et al. (0.053, \( p = 0.090 \)) and our replications, industry specialization has an overall positive association with audit fees. That result holds in a median split on asset size for the smallest half of firms in both Casterella et al. (0.097, \( p = 0.040 \)) and our replications, but there is no statistically significant association above the median in either their table (0.009, \( p = 0.44 \)) or in our replications. When we repeat the test using a finer quintile partition, however, we see that the association between auditor specialization and audit fees has the expected positive sign consistently in only two of the smallest three quintiles, with the second quintile being marginal but not significantly positive. There is no association in Quintile 4, while the association in the largest quintile of clients actually takes a strongly negative sign.
**Figure 1a** Audit fees vs. total assets (fees <= $20 million and assets <= $60 billion)

**Figure 1b** Log of audit fees vs. log of total assets (fees <= $20 million and assets <= $60 billion)
**Figure 2**  The relationship between fees and other common non-negative continuous independent variables

**Figure 2a** Elasticity of fees to quick ratio by quick ratio quintile

**Figure 2b** Elasticity of fees to debt to assets by debt to assets quintile

**Figure 2c** Average log of fees by quick ratio decile

**Figure 2d** Average log of fees by debt to assets decile
Notes:
Figure 2 shows the relationship between fees and other common continuous, non-negative predictor variables. Figures 2a and 2b show the average elasticity of fees to quick ratio and debt to assets based on yearly regressions by quintiles of the respective variable, using the logged variable as the regressor (Model 1 quintile results in Table 4). The figures also show the estimated elasticity for each quintile based on yearly pooled regressions using the respective unlogged variable as the regressor (using the Model 2 results in Table 4), the common methodology in audit fee estimation. Linear and polynomial estimations are provided for the Model 1 quintile results and a linear estimation is provided for the Model 2 results. Figures 2c and 2d show average logged audit fees as a function of quick ratio and debt to asset deciles, respectively. The patterns in the graphs clearly correspond to the Model 1 quintile results in Figures 2a and 2b.
Figure 3  Log of audit fee vs. student residual (simple regression)
**Figure 4a** Mean regression residuals by company size - full sample versus quintile and decile estimations

**Figure 4b** Mean regression residual by company size - full sample versus median split estimations