Optimal procurement portfolios when using B2Bs: A model and analysis

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Abstract

B2Bs are online markets where buyers and sellers trade products either in the spot market or via derivate instruments such as option contracts. Our goal in this paper is to show how procurement managers, in addition to buying on the spot market in cash (physical transactions), can integrate risk management tools (paper transactions) to mitigate risk over multiple time periods. Specifically, our scenario includes a buyer who is making procurement decisions to satisfy demand. Demand can be satisfied either by buying and exercising options on the B2B (for a price) or by directly trading on the spot market with an uncertain spot market price distribution. Over a two-period time horizon, we provide a model to compute the optimal number of options and physical quantities of the product that are needed to satisfy demand while minimizing relevant procurement and inventory costs over the two periods.

1. Introduction

Over the last decade, online trading exchanges (“B2Bs” from now on) for certain non-commodity products like chemicals, semi-conductors, or bandwidth have provided firms with a new channel to trade: either in the “spot” market or via derivate instruments such as options, futures, or forward contracts. On one hand, it has given procurement managers an alternative procurement channel to alleviate the demand risk from traditional procurement channels. On the other, despite derivative instruments to alleviate price risk, B2Bs also expose procurement managers to price volatility. Our goal in this paper is to build a model to illustrate how procurement managers, in addition to “physical transactions” on the spot market, can integrate a B2B’s risk management tools such as the option to buy (“paper transactions”) into their procurement decisions, primarily to hedge against price volatility.

1.1. Problem setting, spot markets, and options

Consider a procurement manager of a price-volatile product whose primary objective is to minimize procurement costs for her division. One sure way to protect margins is to sign a long-term contract with stable prices (for example, when buying a product like Benzene, these long-term prices are typically tied to an index like Platt’s). This has a couple of primary disadvantages: when market prices are relatively lower (all else being equal), margins are potentially lower. Secondly, price-volatility does not limit exposure to physical or in-transit inventories. B2Bs provide a viable solution to reduce costs not only by creating competitive spot markets, but also by providing risk management tools such as the “option” to stabilize margins. In fact, the very same suppliers who provide long-term contracts are setting up B2Bs to sell the product in cash or options to their customer (with the intent of increasing their profits, of course).
A practical approach, however, is for procurement managers to use both long-term contracts with preferred suppliers (these suppliers have a close relationship with the buyer’s company and often participate in supply chain initiatives) and procure a portion of the demand (say $D$) from the B2B. In this paper we concern ourselves only with that portion of the demand ($D$) that is placed in the B2B. Specifically, we consider a two-period problem with the objective of satisfying a demand $D$ in each period from the B2B at the lowest possible procurement cost. To protect against the unpredictability of margins in a spot market (for example if spot prices tend to be very high relative to the long-term contracts), the manager also protects herself by buying options.

In our problem context, an option is a customized contract where the buyer agrees to buy the product at a pre-determined price and time period (an excellent review of financial derivatives is available in Hull, 2000). The typical procedure is for the manager to buy (go long) option contracts into the future, and when the product is needed, buy it in the cash/spot market and sell the options simultaneously. Thus, a buyer of an option contract will be able to procure units at a pre-determined price (if she decides to exercise the options), irrespective of the actual spot price on the B2B.

In our two-period model scenario, the buyer buys options at time $t = 0$ which are executable either at time $t = 1$ or 2. At time $t = 1$, after observing the spot price, a decision is made (a) either to buy at the spot market price or (b) execute options to satisfy demand $D$. At time $t = 2$, demand is satisfied by a combination of inventory (if any is carried over) and from either spot market or option contracts (if they were not executed at $t = 1$). Our model finds the optimal options to buy at time $t = 0$ and the quantity to procure at the spot price at the time $t = 1$ (all other decisions in the model resolve from these two) to minimize cost.

Our model builds on three streams of literature. First, there is an extensive body of literature on the multi-period lot-sizing problem (see Gupta and Keung, 1990). Motivated by the basic Wagner-Whitin formulation, researchers have extended it to include multiple products, plants, resource constraints (such as capacity and budgets), but few, if any, deal with spot markets and option markets together.

Our model also builds on a stream of research within the contracts literature in supply chain management that deals with using multiple sources of supply. Much of the research on supply chain contracts (see Tsay et al., 1998 for a review) either derive optimal inventory and distribution policies given the construct of a contract, or provide guidelines for contract parameters, given the operating policy of the firm. The results from this stream of research indicate that firms typically choose short-term contracts unless the long-term contracts provide for a threshold cost improvement over the planning horizon (Cohen and Agrawal, 1999). Secondly, firms can successfully use the spot market to improve margins (Akella et al., 2002).

Finally, there is a growing stream of literature that models contracts on B2Bs (see Kleindorfer and Wu, 2003; Spinler et al., 2003). Most papers in this stream use game theory to model the trading mechanism between buyer and seller on the B2B. The typical objective in this stream is to seek out and analyze the best contracts that optimize utility of either buyer or seller or both. This paper is perhaps closely related to Aggarwal and Ganesan (2007) who consider a single period problem and find the optimal procurement portfolio—the number of units to procure from a primary supplier; the number of forward contracts to write; and the quantity to buy at the spot market—to satisfy stochastic demand.

We contribute to the literature in two ways: (1) we add to the models that integrate the physical and paper trades to minimize costs; and (2) we do it over a multiple period time horizon. The remainder of the paper is organized the following way. Section 2.3 describes the model; Section 3 explores the optimal solution with the numerical example and Section 4 summarizes the paper with directions for future research.

### 2. Model development

#### Notation

- $D$ Demand in each period
- $X$ Cost of an option
- $\delta$ Premium (or discount) charged per option
- $C$ Option strike price
- $p$ Random variable denoting spot price with pdf $f(p)$ and cdf $F(p)$. We also assume that the spot price distributions in period 1 and 2 are independent. To improve clarity, we just use $p$ to denote spot market price in either period 1 or 2 and let the context identify the period.
- $\mu$ Mean spot price
- $h$ Unit holding cost per time period
- $S$ Delivery charge
- $Ns$ Expected number of deliveries
- $ETC$ Expected total cost of procurement

#### Decision variables

- $O$ Options purchased at time $t = 0$
- $Q$ Quantity purchased on the spot market at time $t = 1$

#### Assumptions and problem setting

- The members of the supply chain are the procurement manager (“buyer”), a supplier, and a B2B exchange where they can trade either options or in the spot market.
- The buyer has to fulfill demand for two time periods, $t = 1$ and 2. We assume that the demand for each period is $D$, a constant. The buyer may decide to procure more than $D$ at time $t = 1$ and inventory any units in excess of $D$. However, no backorders are allowed at the end of period 1.
- At time $t = 0$, the buyer makes a decision to buy $O$ options each costing $X$, executable at a price $C$ at either
t = 1 or 2. If the buyer decides to execute the options in a period, (s)he must execute all O options. This paper uses the following decision rule to decide when to exercise the options: if the buyer has options available and observes a spot price higher than the strike price C in either time period, the options are exercised.1 Options not exercised will expire at the end of period 2.

- At time t = 1, the buyer’s objective is to satisfy demand either by executing options (if the spot price is less than C) or by procuring Q ≥ D in the spot market. We assume that when the buyer buys Q units from the spot market, enough is purchased so that together with the options at hand, it is enough to satisfy demand for both periods, i.e., Q + O ≥ 2D. As the next section will show, depending on the specific values of Q and O, there are several possibilities on how the demand can be satisfied.

- At time t = 2, the buyer will satisfy demand from inventory carried from period 1, and procuring the remainder by either option contracts (if any available) or at the spot market.

2.1. Enumerating the expected total cost (ETC)

Based on the number of options that are carried over into t = 1, following are enumerations of the ETC:

\[
\text{ETC} = N_s S + Q \mu + (Q - D) h + (2D - Q) \mu, \quad O = 0 \quad (1)
\]

\[
= XO + N_s S + \int_{P < C} [Q p + (Q - D) h + (2D - Q) E(\min(p, C)) f(p) dp + \int_{P > C} [O C + (D - O) p + D \mu f(p) dp, \quad 0 < O < D \quad (2)
\]

\[
= XO + N_s S + \int_{P < C} [Q p + (Q - D) h + (2D - Q) E(\min(p, C)) f(p) dp + \int_{P > C} [O C + (O - D) h + (2D - O) \mu f(p) dp, \quad D \leq O \leq 2D \quad (3)
\]

Eq. (1) represents the case when no options are bought at t = 0. The buyer satisfies demand by simply buying on the spot market. The first term is the delivery cost and Ns, as the ensuing discussion will show, is the expected number of deliveries. If Q = 2D, Ns = 1 else Ns = 2. In the first period, the buyer will procure Q units, and inventory (Q - D) units at a cost of $h$ per unit. Any remaining demand in period 2 (2D - Q) will be satisfied by procuring from the spot market (at a mean price $\mu$).

Eq. (2) represents the case when the buyer has purchased $0 < O < D$ options in t = 0. The first line of the equation is the sum of the expected cost of options and delivery charge. The second line of Eq. (2) represents the case when the spot price, $p$ is less than the strike price $C$ at t = 1. Per our decision rule, the buyer buys Q units from the spot market where Q ≥ D. Any units in excess of demand Q - D is inventoried at a cost $h$ per unit. Any remaining demand in period 2, (2D - Q), will be satisfied at the prevailing spot market price or the strike price which ever is smaller. The third line of Eq. (2) represents the case when C is less than the spot price in period 1. The buyer will therefore execute her options. OC is the procurement cost of O units. Since there are not enough options to satisfy demand, (D - O) units are bought from the spot market to satisfy demand. The demand in period 2 will simply be satisfied by procuring the remaining demand D from the spot market at an average price of $\mu$.

Eq. (3) represents the case when D ≤ O ≤ 2D options are purchased at t = 0. The interpretation of the first and second lines of Eq. (3) is the same as Eq. (2). The third line is the case where the observed spot price in t = 1 is greater than C. The options therefore will be executed in t = 1. OC the cost to procure the units in t = 1 at a price C. Since O > D, (D - O) units are now held in inventory at a cost of $h$ per unit. Any remaining demand in t = 2, (2D - O), will be procured from the spot market at a mean price $\mu$.

Eqs. (1)-(3) are discontinuous due to the fact that $N_s$, the expected number of deliveries, changes depending on the specific value of Q and O. $N_s$ is given by

\[
N_s = \begin{cases} 
1, & Q = 2D, \quad O = 0 \\
2, & D < O < 2D \quad 0 < O < 2D \\
1 + F(C), & O = 2D, \quad O = 2D \\
1, & Q = 2D, \quad O = 2D 
\end{cases} 
\]

When Q = 2D, only one delivery is needed if the procurement portfolio has either O = 0 (this is the case where the buyer does not hedge) or when O = 2D (all buying decisions in period 1 procure 2D units). On the other hand, the range D ≤ O < 2D and 0 < O < 2D represents the scenario where some buying occurs in either period—so two deliveries are required. When Q = 2D and O < O ≤ 2D. One setup is required if the spot price in period 1 is less than C, else two are required, hence the average deliveries are 2 - F(C). Similarly, when D ≤ Q < 2D and O = 2D, one setup is required when the spot price in period 1 is greater than C, else two are required making the expected value 1 + F(C).

2.2. Pricing options

When the buyer buys options priced at C in an uncertain spot market $p$, the effective price of procurement is $E(\min(p, C))$, which is less than $\mu$, the mean spot price. A fair value of the option price for both the buyer and the supplier trading on the B2B is to make it equal to the intrinsic value of the forward $X = \mu - E(\min(p, C))$ (see Aggarwal and Ganeshan, 2007). $X$ can be interpreted as the cost where the buyer is indifferent to writing options or making trades in the spot (cash) market. Often the supplier may want to include a premium (or discount) $\delta$ when pricing options to reflect other market conditions. For example, $\delta = 0$ will represent the fair value, $\delta > 0$.
represents the case where the options are sold at a premium (a case where the market is expected to make a fast upward move), and \( \delta < 0 \) is the case where the options are sold at a discount (a case where the supplier offers an incentive to buy options for the future in the hope to stabilize his/her profit margins).

\[
X = \mu - \left[ \int_{p \leq C} pf(p) \, dp + C(1 - F(C)) + \delta \right] \tag{4}
\]

2.3. Optimal solution properties

1. The optimal \( Q \) will either be \( D \) or \( 2D \).
2. When \( \delta \leq 0 \), the optimal \( Q \) will be either \( D \) or \( 2D \).
3. When \( \delta > 0 \), the optimal \( Q \) is either 0 or some small number \( \epsilon > 0 \).

**Proof.** Let \( \beta = \int_{p \leq C} pf(p) \, dp \) and \( \theta = \int_{p > C} pf(p) \, dp \).

Eqs. (1)–(3) reduce to (5)–(7):

\[
\text{ETC} = N_{S} + 2D\mu - Dh + Qh \quad Q = 0 \tag{5}
\]

\[
= N_{S} + 2D(\mu - \delta - X)F(C) - hF(C) + \mu(1 - F(C) + \theta) + Q(\beta + F(C)(h - \mu - \delta + X)) + O\delta \quad 0 < O < D \tag{6}
\]

\[
= N_{S} + 2D(\mu - \delta - X)F(C) - hF(C) + (2 \mu - h)(1 - F(C)) + O(\mu - \beta + (h - \mu)(1 - F(C)) + \delta) \quad D \leq O \leq 2D \tag{7}
\]

1. From (5)–(7), ETC is linear in \( Q \), for \( D \leq O \leq 2D \). For a given \( O \), if ETC is increasing in \( O \) for \( D \leq O \leq 2D \) (i.e., the marginal cost of buying an extra unit at the spot market in period 1 is positive), then we just compare ETC at \( Q = D \) and \( Q = 2D \), since \( Q = 2D \) may require one less delivery and can be potentially cheaper. On the other hand, if for a given \( O \), if ETC is decreasing in \( Q \) for \( D \leq O \leq 2D \) (i.e., the marginal cost of buying an extra unit at the spot market in period 1 is negative) then the optimal solution will be \( Q = 2D \), since \( Q = 2D \) is a special case of ETC in \( D \leq O \leq 2D \) with the added benefit of potentially one less delivery.

2. When \( \delta = 0 \) (i.e., the options are priced at their fair value), Eq. (6) becomes independent of \( O \), i.e., the buyer is indifferent to buying \( O < O \leq D \) options. A practical solution when \( \delta = 0 \) is to buy \( D \) options. When \( \delta < 0 \), ETC is decreasing with increasing \( O \) for a given \( Q \) in \( 0 < O \leq D \). So the manager will buy at least \( D \) options.

From (6) and (7), ETC is piecewise linear in \( O \), for \( 0 < O < 2D \). For \( \delta < 0 \) and a given \( Q \), if ETC is increasing in \( O \) for \( D \leq O \leq 2D \) (i.e., the marginal cost of adding an option is positive), then the optimal solution is either \( O = D \) or \( 2D \), since \( O = 2D \) may require one less delivery and can be potentially cheaper. If on the other hand, ETC is increasing in \( O \) for \( D \leq O < 2D \), the optimal solution is \( O = 2D \) since \( O = 2D \) is a special case of ETC in \( D \leq O < 2D \) with the added benefit of potentially one less delivery.

3. With \( \delta > 0 \), Eq. (6) is increasing in \( O \). If \( \delta \) is large enough to make ETC in Eq. (5) smaller than Eq. (6), the optimal solution is \( O = 0 \). However, if \( \delta \) is not large enough, buying a small number of options \( \epsilon > 0 \) will always be cheaper. The cost of every additional option is \( \delta \), so a practical approach would be to hedge some pre-determined minimum amount in options (since this will be cheaper than not hedging anything). □

3. Numerical illustration

As a simple example to illustrate the interplay between buying options and trading in the spot market to satisfy demand, we consider the case where \( p \) is uniformly distributed. Fig. 1(a) shows the ETC for \( D = 100, h = 5000 \), \( S = 50\,000 \), \( D = 5 \). From this, the fair value of this option is \( S(21.125) \). Fig. 1(a) shows the structure of ETC wrt to \( Q \) and \( O \). The optimal value occurs at \( D = 200 \) and \( Q = 100 \), i.e., the buyer at \( t = 0 \) will buy 200 options with a strike price of \$85 costing \$14.125 each, and in \( t = 1 \), if the spot price is lower than \$85, will buy 100 units.

Figs. (b)–(d) show the sensitivity of the optimal solution to \( h \), \( S \), and \( \delta \), respectively. As the holding costs increase, the optimal \( Q \) and \( O \), as expected, also decrease. In this scenario, as \( h \) increases to \$3.63 from 0, optimal \( Q \) is reduced to 100 from 200 units and as \( h \) further increases to \$43.27, the optimal \( O \) changes to 100 from 200 units.

Fig. 1(c) shows how the optimal procurement changes with the delivery charge. As the delivery charge increases, \( Q \) and \( O \) (again as expected) also increase. When the delivery charge is relatively high (around \$10,000 in the chart), the optimal strategy is to hedge the entire demand through options and buy the entire demand at spot if the spot price in \( t = 1 \) is less than \( C \).

Fig. 1(d), meanwhile, illustrates how \( Q \) and \( O \) change with \( \delta \). It is easy to see that the maximum discount is just the fair value of the option (so in this case if \( \delta = -21.125 \), \( X = 0 \)). When \( X = 0 \), the optimal solution is \( O = 200 \) and \( Q = 100 \). As \( \delta \) increases in value to \(-48.82\), the optimal \( O \) reduces 100 units. When \( \delta > 0 \), a small number \( \epsilon > 0 \) will be optimal (the chart does not indicate this small quantity). This simply means that buying options will be cheaper than not buying options at all. Each additional option increases ETC by \( \delta \)—so depending on the value of \( \delta \), a pre-determined number of options (\( \epsilon > 100 \)) can be purchased at \( t = 0 \).

4. Summary and conclusions

In this paper, we provide an exploratory model that helps a procurement manager determine the right combination of paper trades (options) and physical transactions (spot market) that will minimize the total expected cost of procurement over two time periods.

Specifically, we assume the demand \( D \) in each time period is constant and the total demand over the two time periods is satisfied with a combination of \( O \) options purchased at time \( t = 0 \) and the quantity \( Q \) procured at the spot market (if the spot price in period 1 is less than \( C \)). The optimal combination of \( O \) and \( Q \) minimizes the
expected sum of purchase, delivery, holding, and the cost of writing options.

Our model provides one way to integrate paper with physical transactions over two time periods. The model gives procurement managers and policy makers guidelines on how to use the spot market and the risk management tools B2Bs provide. Depending on the product characteristics, the optimal procurement strategy will change. For example, when setup costs are relatively high and option premiums low, the optimal solution will be to buy enough to satisfy the entire demand from either the spot market or options (the planner will ultimately choose which ever is cheaper). If on the other hand the holding costs are high, spot and option quantities are tailored so the managers will not have to hold inventory yet satisfy demand while maximizing margins. Following are suggestions for future research:

1. We used a two-period problem with simple decision rules for option execution to reduce the complexity of the problem. A natural extension would be to extend it to multiple time periods and to also consider the case of stochastic demand. The problem will be very complex since the manager now has the ability to use either options or the spot market in any period and hence the number of possible ways to satisfy demand will be large.

2. There is a need to integrate the various derivate instruments available in B2Bs such as forwards, swaps, and options seamlessly with more stable procurement channels that rely on long-term contracts.

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References


